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A Proposed Entropy Measure for Assessing Combat Degradation

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This paper describes a method for assessing the combat degradation of an army, based on the Shannon entropy measure. It is shown that this measure is useful for analyzing combat degradation due to casualties, since it is in broad agreement with the values established from past battle experience. It is also shown that the measure can be used to evaluate the degree of confidence in the outcome of training maneuvers when weapons simulators are employed.

Keywords: analysis, defense, entropy

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INTRODUCTION

We consider the problem of calculating how many weapons simulators are required for an exercise involving a fighting force of N combatants when a certain number to of casualties is expected. Shannon's entropy, representing the average amount of information[1] in an army, is useful in this regard. Changes in this measure over the course of a battle can help in the assessment of combat degradation due to casualties.

The comparison between the values of the entropy of an army, as calculated by the method presented in this paper, and historical data for the loss of effectiveness in actual combat[2] shows good agreement. It is also found that the very same concept of entropy can be used to determine the effectiveness of weapons -simulators deployed in a military exercise.

ASSESSING MILITARY MANOEUVRES

Military exercises or maneuvers serve two purposes. First they are designed to provide training for individual servicemen; and second, they are intended as a basis for training and testing systems of command. These two aspects require high-resolution models, when the fate of individuals is accounted for, and aggregate models, taking large units as the basic elements,[2,3] respectively.

The problem with both theoretical models and live training maneuvers is centered on the validation of their results, i.e. the certainty as to what would be the true outcome if the exercise were a real confrontation.

The validation procedure is normally based on information recorded throughout the exercise by experienced military referees, backed up by standard battle charts.[4] Nevertheless, the decisions and the lessons to be extracted from the exercise are sometimes subject to controversy, because they have in themselves a degree of uncertainty.

The advent of weapons simulators attenuated this problem, because the degree of uncertainty decreases when such simulators are distributed to the participants in the exercise.

In the limit, if every single weapon could be simulated precisely as regards its effects, then the exercise-except for the obvious psychological aspects of battle[5] - would show the same results as those of a real battle. But simulators are expensive, and it is impracticable to equip every participant in a large exercise with this kind of facility. Hence, it is important to assess how many simulators to deploy, while retaining a high degree of confidence in the realism of the exercise and in its military outcome. For this purpose, it is proposed to use an entropy measure derived from the Shannon entropy formula.

THE ENTROPY OF AN ARMY

The characterization of an army is traditionally done by two sets of parameters of a very different nature. First, a quantitative set consisting of the number of men, the number of weapons, their rates of fire and fire-power density-all of which are additive variables. The other set contains qualitative variables concerning morale, the state of readiness, etc. Since these are difficult to measure, most models are restricted to the first set of variables.

Existing combat models work within a mechanistic point of view. The evolution of combat situations is described by a set of parametric differential equations, depending on the different types of weapons employed. The validation of these models is done by curve-fitting with historical data, mainly from WW II battles.[3]

To describe an army with such a mechanistic model requires the handling of a very large set of variables and an extremely large amount of data. But, apart from all the practical difficulties, there is an ever-present problem connected with the validity of a mechanistic description based upon sets of differential equations, when a battle is typically an attrition or a dissipation phenomenon. Under these conditions, an army can be better modeled as a thermo dynamical system.[6] In particular, the entropy of an army can be used to ascertain the level of degradation of its combat capacity.

According to Shannon,[7] the entropy H of an event occurring with probability p is given by

$$H = \log 1/p \qquad (1)$$

In the case of an army with N combatants, if there are σ casualties, then the probability of each casualty is given by σ/N. This means that, for each casualty, the contribution to entropy is

$$H(N, \sigma) = \log N/\sigma \qquad (2)$$

Hence the Shannon entropy of the whole system is given by

$$\langle H(N, \sigma)\rangle = (N/\sigma).\log N/\sigma \qquad (3)$$

Multiplying the Shannon entropy in equation (3) by N gives the equivalent of the Boltzmann measure of entropy, which can be written

$$\langle S(N, \sigma)\rangle = \sigma.\log N/\sigma \qquad (4)$$

Equation (4) corresponds to the usual measure of entropy in thermodynamics. Therefore, if this measure applied to combat degradation is found to agree with the values established by past experience, then we might be justified in using it to measure the loss of military effectiveness due to casualties.

ENTROPY AND ASSESSMENT OF COMBAT DEGRADATION

A study of casualties has indicated that the effectiveness of an army reaches a critical level when the number of casualties lies between 25 and 35% of the number of combatants[2] Beyond this point all efforts, no matter how heroic, will not be able to avert defeat. That means that a parameter which is useful for measuring combat degradation due to casualties must peak at around these percentages.

Figure k depicts the evolution of the army's entropy computed from equation (3) as the percentage of casualties increase. The main result is that entropy reaches its maximum at 37% of casualties. Also, with about 10% of casualties, the entropy is about 60% of its peak value; and it reaches 84% and 92% of its maximum when the casualties attain 20% and 25%, respectively. Figures quite similar to these are accepted in the existing attrition models for the loss of combat effectiveness of an army.'

Thus the behavior of the entropy given by equation (4) is close to what past experience has shown to be the degradation of the combat capacity of an army due to casualties.

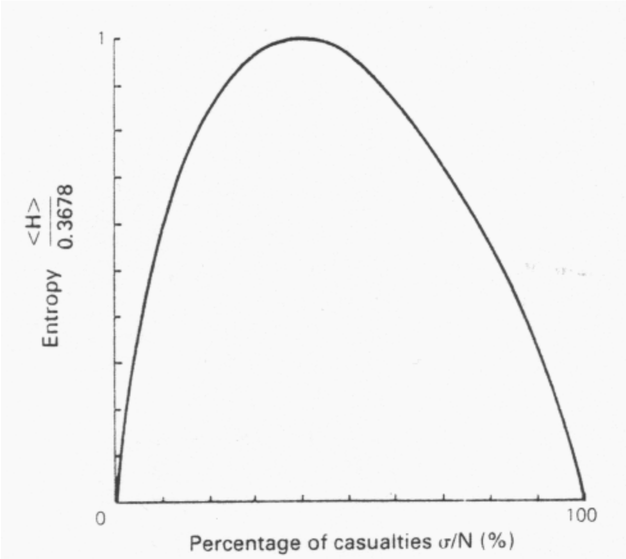


FIG. *1. Entropy and percentage of casualties.*

WEAPON SIMULATORS AND THE SIMULATION PARAMETER

In a normal training maneuver, casualties are just not produced. But if a percentage k of weapon simulators are deployed, the Boltzmann entropy per simulated casualty, is given by

$$L(N, \sigma, k) = \log (N/ \sigma.k) \qquad (5)$$

Hence the ratio

$$\Sigma ((N, \sigma, k) = H (N,\sigma) / L((N, \sigma, k) \qquad (6)$$

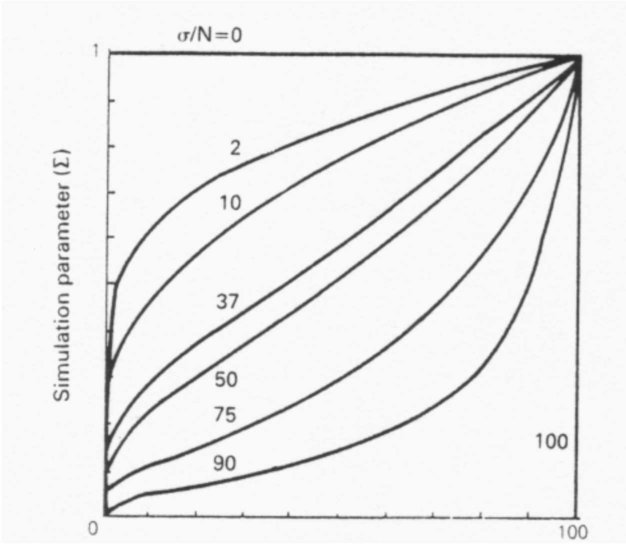
measures the degree of confidence in the results of an exercise with *kN* simulators when, in real conditions, the army of *N* combatants would be expected to have σ casualties. Σ *(N,, σ, k) will* be called the simulation parameter.

Simulation parameter and degree of confidence in an exercise

The behavior of the simulation parameter defined by equation (6) is shown in Figure 2. The simulation parameter has a value of zero, indicating ‘no confidence', when there are no simulators present, and a value of one, indicating ‘one hundred per cent confidence’, when, regardless of the number of casualties, every combatant has a simulator. Moreover the simulation parameter is always equal to one when there are no casualties, because in this case any exercise would always simulate perfectly what would happen in reality.

As the number of expected casualties begins to increase, the number of simulators deployed must also increase in order to have the simulation parameter above a certain value. As the number of expected casualties surpasses *50%* of the total number of combatants, the curvature of the simulation parameter curve also changes.

When the expected number of casualties is 100% of the number of combatants, the simulation parameter shows that in order to have any degree of confidence in the way the exercise evolves, and in its final result, it is necessary to give a simulator to every combatant. Hence, these two limiting cases-no casualties and one hundred per cent casualties-demonstrate the consistency of this method. Furthermore, for a given degree of confidence (Σ = constant), the required number of simulators increases as the percentage of casualties rises (Figure 3), in a way which seems quite reasonable. Thus, when setting up a training maneuver, the simulation parameter can be used to



Percentage of simulators K (%)

FIG. 2. *Simulation parameter and percentage of simulators.*

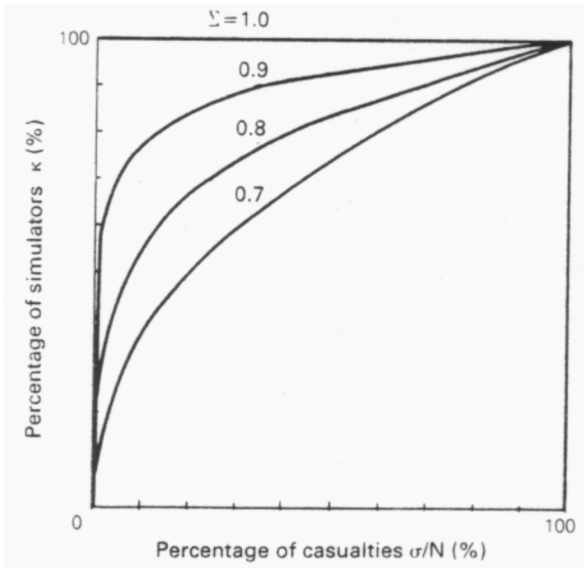


FIG. 3. *Percentage of simulators and percentage of casualties.*

determine the percentage of weapons simulators required in order to achieve a predetermined level of confidence in the outcome.

From Figure 3, it is apparent that, for the level of casualties in which we are interested, i.e. below 37%, the required number of simulators increases very rapidly with the number of casualties.

SUMMARY

The Shannon entropy formula was used to derive an entropy measure of the combat degradation of an army. The maximum of this entropy function at 37% of casualties, and its values for lower percentages of casualties, are consistent with the evolution of the loss of effectiveness of an army due to casualties, as is derived from past experience. Thus, it is proposed that the entropy of an army as defined in this paper could be a parameter to assess combat degradation due to casualties.

The ratio between the entropy per casualty in real combat and the corresponding entropy in an exercise with weapons simulators can provide a measure of the degree of confidence on the results of the maneuvers.

This ratio was called the simulation parameter. For the limiting cases of no casualties and one hundred per cent casualties, the values of this parameter showed their own intrinsic consistency. The simulation parameter is valuable for determining the number of simulators required for a particular exercise when a certain degree of confidence in the results must be reached.

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