Models of Combat with Embedded C2 VII: Casualty Based Entropy Calculations as a Combat Predictor;

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Abstract:

Entropy as a measurable quantity, *derivable from, combat data, is* introduced as *a global concept by which to express the disorder of combat entropy is done using ,historical combat data. Both static tend time series has been been employed. The use of entropy as a predictor for use in the design of C2 systems is explored. A state space diagram based on entropy values related to combat outcomes is introduced in this regard. Comparison of the entropy computations with the results of power spectral analysis of the same data is also introduced.*

Background LO the *Models* Series

The subject of modeling combat with embedded command and control (C2) is being systematically developed in a continuing series of papers, which we refer to here as the *Models* Series (1988-90). A wide range of treatments spanning applications of catastrophe theory, cellular automata, and system dynamics have already appeared. All contributions to the series are threaded with a common conviction that C2 theory can not be *developed in* the abstract, divorced from a description of combat. That conviction is borne out in this paper which deals with entropy computations derived from combat data.

Introduction

We have been guided in this series by a number of propositions. Perhaps foremost among them has been the assumption that combat is characterized by local chaos and long range order. The local chaos is often deliberate as the goal of either combatant is to sow disorder while preserving his own structural integrity. We have hypothesized that combat with embedded C2 is a self-organizing system with training and discipline playing a major role in the aforesaid process. Thus, it is our contention that it is C2 which serves to give structure to combat. It has been almost a tenant of faith among commanders that infliction of casualties reduces the structural cohesion of a force, and in turn sows the kind of disorder that presages collapse on the battlefield. collapse on the battlefield. Measurement of that disorder has proven elusive although there is a theoretical quantity of such is by definition a measure of disorder. That quantity is entropy.

In this paper we shall explore the consequences for CG of a recent proposal by the senior author that entropy, computed from casualty reports is a predictor Combat outcomes (Carvalho-Rodrigues, 1989). If this proposition can be substantiated, and we shall seek to show from historical data that it can, then entropy should prove a means through which to evaluate C2 effectiveness. 1 paving advanced that notion, one turns automatically to the possibility that entropy may serve as a predictor when extracted from 3n accumulating time series. If entropy has predictive value for C2, then its evaluation can be instrumented from incoming reports of casualties.

Our Case will be advanced as follows. We will begin with a necessary review of the entropy concept in a number of guises. Then the rather surprising notion is advanced that entropy is *unifying* principle. A Shannon type entropy equation is then tested against selected historical data. A case for prediction is advanced. Limited time series casualty data is also analyzed with excellent results. Finally, the time series data is also subject to power spectral analysis which indicates that the data is primarily from a single wave attack.

Entropy

The Basics

It is our contention that far from being an obscure concept, entropy is a global, and measurable, parameter which is particularly appropriate for characterizing systems. In this interpretation entropy is considered as basic to the parameterization of systems as is mass to the system's physical component. Entropy is a macroscopic or extensive property. On this assumption entropy will tell us something about basic changes in the systems while Ignoring details of internal systems interactions.

Measurement of overall system entropy should be as straightforward as the measurement of the mass of components for systems that ha'/e physical components. It is not; perhaps because entropy is usually not computed. We note in passing that the general question of basic parameters for system characterization we received little attention with the possible exception of complexity parameters 1989). What becomes important to us is to pick a form of entropy which s appropriate to combat viewed as a system. We would also call attention to our investigations into the fractal nature of combat (See Models VI, in tile Models Series). It appears possible that data on hierarchical systems fit a hyperbolic rower law. This finding leads us to believe that the racial dimension from such a fit may be another example of system variable which is insensitive to the details of a particular structure.

In a theoretical sense the concept of entropy is commonly associated in some fashion with the disorder present in a system. One usually encounters entropy for the first, and last time, in the study of thermodynamics where it enters in the expression for the second law of thermodynamics. In the second law entropy (S) is related to heat (Q) and temperature (T) as $dQ/T = dS$. Maxwell's thermodynamic equations make extensive use of the entropy concept. If asked what they remember about entropy, the average engineer would probably answer that it keeps growing; and that it is not very useful. That putative conclusion reckons without the extension of the entropy concept to information theory by Shannon. He expressed the equation for noise in an, information channel as:

$$H_S = - p \ln p, \text{ where p is a probability.} \quad (1)$$

Unlike the thermodynamic entropy S, the value of HS reaches a maximum and then declines. Figure 1 shows this generic behavior.

Figure , Goes Here--[Graph showing the dependence of Hs on the value of the probability p].

Relation to Combat

Carvalho-Rodrigues (1989) has exploited the shape of he curie n Figure 1 ;o predict success in simulating battle outcomes by relating casualty production to Shannon entropy. His work assumes that casualty counts (Ci) can be relates to a provability expression (Ci/Ni), where i represents either the Blue or Red force, and where N is the force strength. Rewritten in these terms equation 1 becomes:

$$Hs = (Ci/Ni) \ln (1/(Ci/Ni)) \quad (2)$$

In general we have that Ci = Ci(t), and likewise for Ni. Our main taste in this paper will be to test the consequences of equation 2 against historical data.

What is noteworthy about the curve in Figure 1 is that it passes through a maximum and then declines. The peak is about 37% of p = Ci/Ni. Equivalently stated, we find that although the casualty production may continue past the peak, the chosen measure of system disorder (HS) has passed its maximum. It is as if the 'carrying capacity' of the (combat) system described *by* Equation 1 declines, signifying disintegration of the system itself. Other casualty production benchmarks in terms of the peak value are approximately as follows:

10 % at 60% of peak; 20°,6 at 84% of peak; and 255% at 92°,ó *of* peak.

As discussed *by* Carvalho-Rodrigues (1989), this kind of general behavior is in accord with the expected interaction *of* combat casualties and the breakup of a fighting force as a function of percent casualties sustained. A unit with casualties of 20-30% has endured very heavy casualties indeed. If entropy is to be a valid predictor of combat outcomes, then the data should approximate a portion of such a curve as in Figure 1. For final outcome statistics only, the results over many battles should be so distributed. Time sensitive data should generate only the early portions of such a curve since breakdown, and battle termination, will usually occur before the peak of the distribution.

We may note in passing that for a dissipative system the evolutionary criteria for the generation rate for internal entropy production is given as $dPi/dt \leq 0$ where $Pi = dS/dt$ (Schneider, 1988). In the *Models* Series we have argued consistently that combat is a dissipative system, and in fact a bizarre kind of ecology. Thus it begins to appears only reasonable that entropy production should finally emerge as a predictor of combat evolution, and hence a major contributor to C2.

Other Facets

Closely related to the Shannon entropy, and a generalization of it, is the Renyl entropy expression:

$$H_G = (1/(1-G)) \ln (\sum I\, pi^G) \qquad G > 1 \qquad (3)$$

where G is a kind of system, gain coefficient. H_G has been computed for chaotic systems. A classification scheme for such systems may be related to this property.

Equation 3 does not exhaust the possible forms that entropy may assume. in the early seventies deLuca and Termini (1972) introduced the idea of a ;tizzy entropy in the following form

$$HF = \mu(x) + \mu\,(1 - x) \qquad (4)$$

where μ is a fuzzy membership function. The consequences for command and control have been discussed by Dockery (1982), where a curious property of fuzzy entropy is explored. The property in question predicts that past a certain point HF car, only be lowered by reformulating the hypothesis for which the fuzzy entropy has been calculated. The consequences for C2 are almost obvious. There are predicted to be times when collecting more information about a particular hypothesis fails to produce additional clarification! The foregoing suggests that a hypothesis about whether a side is winning or losing, based on Shannon entropy and contains fuzzy data, must be tempered (adjusted) by the ramifications of Equation 4. The possible formalization of such a proposal was not further pursued in preparing this contribution.

In a recent text Ruelle (1 989) ties the concept of entropy to the currently active topic of chaotic evolution in a manner which treats the statistical analysis of time series for deterministic non-linear systems. While we do not believe for the moment that combat is necessarily deterministic, such aspects have been demonstrated in the attrition process. Moreover, we have hypothesized in the *Models* Series that combat is a chaotic dynamical system of great complexity. Ruelle treats the Kolmogrov-Sinai invariant in chaotic dynamics. This invariant measures the asymptotic rate of information production, and is identified with entropy. Information is created as a system evolves. In our case we have system devolution corresponding to the creation of casualties. Connection of entropy with a system invariant is in accord with our previous remarks about entropy as a systems' analogue of ,'physical parameters like amass.

Still other connections ;ay be made. In *Models* V, we have used cellular automata to model combat. In that paper, a Manchester equation hypothesis was successfully fit to output generated by a series Of automata simulations. Appropriate entropy for cellular automata may be defined. Casti (1989) has introduced a 'topological' entropy, which is defined as a measure of the likelihood of a particular sequence of cells will be produced when starting from a random initial configuration. A companion 'measure' entropy is also introduced to give the probability that one of the configuration possibilities under topological entropy will occur. We conclude from this that a further formalism may be available for estimating entropy of combat when modeled with cellular automata. A discussion of temporal and spatial entropies related to the representation of combat as cellular automata has been provided by Woodcock, Cobb, and DePace (1989).

We turn now to a description of our investigation of combat-related entropy by introducing the data sets that we used in our analyses.

The Data

Overview

Computations of Shannon entropy values (HS) were performed on:

• Time independent (or combat outcome) casualty data, which consisted of profile data on battles for which the only casualty information consists of a tally of the initial forces and total casualties on each side, but not the temporal variations in force strengths during the battle.

. • Time dependent casualty data, which was obtained from several sources including: the historical record; a field exercise and a JAWS simulation of that exercise; and a Lanchester equation-based combat simulation.

Time Independent Combat Cases

in order to test the ability of the entropy measure Hi (i=Red and Blue) to predict Combat outcomes data derived from a series of historical battles assemble by Dupuy for Helmbold of the US Army Concepts Analysis Agency was consulted. (Dupuy & HERO, 1986; Helmbold, 1986). Helmbold (1987) has considered the question of a link between casualties and victory and was also the source of any insightful comments (Helmbold 1989).

The basic data set contains detailed historical descriptions of some 60 battles from circa 1600 until circa 1970. Data on straight forward conflict without excessive maneuver, and preferably in a single assault or meeting engagement was desired. It was hypothesized that if the Hi was a combat outcome predictor then it would have the best chance to manifest itself under the least complicated ground combat conditions. Historical battles without complex C2 were presumed to fit such a description. Therefore, not wishing to introduce additional complexity, the following selection criteria were set which yielded 59 battles satisfying these criteria:

- Under 10,000 combatants per side.
- Battles which lasted up to ten hours.

The data set gives only the final casualty tallies. The list is presented as Appendix A. A few excerpts from this data set are displayed in Table 1.

Table 1: Sample rattles

Battle	Duration (Hrs)	Casualty Ratio-Attack	Casualty Ratio-Defense	Outcome
Culloden - 1746	0.67	1558/5400	309/9000	Defense
San Jacinto-1836	0.30	39/743	1600/1600	Attack
Hill 272-1918	2.5	109/2950	250/2563	Attack
Rawiyeh-1967	4.0	150/5350	300/4350	Attack

The subset favored attackers better than two to one. Three cases that resulted in draws *were* arbitrarily assigned to the nominal defender by a ratio of. In summary:

Mode	Number of Cases
Attack	38
Defense	18
Draw	3

The battles by century are shown below with a further breakout for World Wars I and 11. The absence of battles from WWII is a consequence of our selection criteria

1600's	1700's	1800's	1900's	WWI	WWII
4	13	12	30	23	1

As records duration of combat we have:

0-1 Hr	>1-2 Hr	>2-4 Hr	>4 Hr
13	10	18	18

No further profiting of the data was done. We next discuss the sources of time dependent, or time series, casualty data.

Time Dependent Combat Cases

Obtaining time dependent information on casualty production proved to be a more difficult task. Cur primary input came from some data released to us by t`-3 National Training Center located at Ft. Irwin, California at which military exercises are conducted (Ingber, 1989) Laser firings substitute for live ammunition during such simulated exercises Conditions strongly favor the attacker, who represents an semi-permanent on-site aggressor force.

One set of time profiles of casualties in terms of vehicles destroyed were made available in 5, 10, and 30 minute intervals. The last set was chosen for the most extensive analysis. Smaller time-interval data were used in other, related, analyses. Computations of the accumulating value of the entropy filer based on the initial combat conditions. In addition, the entropy generated in any 5 and 3 minute interval was *JSC* Computed. T he latter was to prove most illuminating as gave us a more synoptic profile of the mutual attacker arid defense r responses. Anticipating results, yet to be introduced, it was observed that the attackers entropy rises suddenly as the attack is pressed then must decline rapidly if the attar is to succeed. The unsuccessful defense signature, by contrast, is a rising entropy that never falls back to low levels signaling defeat.

Excursions to the basic exercise scenario are generated by running through the JANUS simulation, which was originally developed at Lawrence Liver more Laboratories. Six such excursions were also provided in five minute time steps. We used these as well but found they showed somewhat different properties for the engagement process chiefly in terms of the persistence of firings by the defender force after its defeat.

The second sort of time series data came from the use of Operation West Wall conducted near Aachen in early 1945. This campaign was directed at the Siegfried Line and was characterized by heavy Allied reinforcements during the course of the battle. The data, and that for the Inchon campaign discussed below, together with commentary on both campaigns, was graciously supplied to us by Helmbold (1989). Use of information from an actual battle introduces several days of data sometimes with the additional complexity of reinforcements and maneuver. It is necessary to track the battle for a longer time span as the kind of detail available from NTC is not recorded (or recordable) from actual engagements. Daily causality figures from the actual battles were converted to entropy equivalents.

The most complex data used was from an campaign lasting some ?0-21 days (Sept 9 to Oct 4) after the UN landing at Inchon (North Korea) in 1959 during the Korean war. In this case heavy reinforcements characterized the North Korean side but the UN side was reinforced as well at day 9-10. Entropy was computed from dally casualty figures talking Into account reinforcements during the subject period.

Our final source of time series casualty data, or rather pseudo data. came from a simulation developed especially for this work. basically we generated a time history of ;mutual attrition for two sides using Lanchester equations in a manger to be described later on. It was anticipated :hat the 'pure' attrition-based Lanchester solution would provide an idealized data sample. And, as we shall see, we were not disappointed.

Analysis Employed

The Historical Battles

We computed entropy values Hd and Ha (for defender (d) and attacker (a), respectively) using equation 2, but with casualty production normalized to unit time, for all 7-9 battles in the data set. This provided what was fundamentally an averaged entropy production rate. However, a couple of very short battles gave unexpectedly high entropy rates using this technique.

From the computed entropies the quantity δi = (Hd - Ha) was selected as the predictor of the combat outcome. Results were as follows in Table IIa

Table: Ha: Results of Normalized Computation of 5i

Mode	Correct	Incorrect
Attack	35	3
Defense	15	6
Totals	50	9

The results in Table IIa were compared with the hypothesis that the figures could have been generated at random. The chi-square was computed with one degree of freedom for a 2x2 contingency table with a value of $\chi_2 = 25.75$.[3] By comparison the significance level for 55% confidence is 3.84. and for 99% is 5.54

As we have said the values for Hd or Ha above are based on an entropy which has been "normalized" to unit time. The normalization turns into an average rate Of entropy production. Results were somewhat less sanguine when δi = (Hd - Ha) was computed from un-normalized data. Those results are in the Table IIb are

Table Ilb: Results of Normalized Computation of δi

Mode	Correct	Incorrect
Attack	31	7
Defense	15	6
Totals	46	13

The chi-square value for Table IIb was $\chi^2 = 16.25$ which is still above the 99% level

In analyzing the data we discovered that real problems with the predictor can arise for battles with casualties which go on beyond the 37% stage, which is the peak of the p.ln (1/p) curve. This is because the curve is zero at both ends. Thus, an attacking force, which lost but a percent or two, but annihilated the defender would incorrectly be predicted as the loser. To compensate for this phenomenon

3. The displays in Tables II are not in the proper form for a contingency test. The rows must be relabeled and the numbers in the second row reversed to use the standard chi-square formula.

We also computed a f(Hi) (where i=d, a) from the un-normalized area under the curve ire Figure 1. Thus

$$f(Hi) = Zi = \int p \, Ln\,(1/p)\, dp \qquad (4)$$

$$Zi = (p^2/2)\,(\ln(1/p) + 1/2)$$

Table No depicts these results where the quantity N = (Zd - Za) was computed

Table IIc Results of Un-normalized Computation of Δi

Mode	Correct	Incorrect
Attack	34	4
Defense	16	5
Totals	50	9

The chi-square for table IIc was χ^2 = 26.03, the best of *the* three '-v .a small margin.

Whatever results we get from use of any of the three predictors in Tables II, we are faced *with* the fact that the predictive value of entropy lies to 'he left side Of the peak value of tile (p.ln (1/p)) curve. This conclusion will be substantiated in Figures 3 shortly to be introduced.

It is to be noted that small differences in attacker and defender entropy predominate in the normalized case. This can be seen in Figures 2 where histograms both 8i and for -Ai are displayed

[Figures 2 Go Here--Histograms of the quantity (Hd - Ha) or (Zd - Za)]

The predictions were better for battles past about 1850. Some of the early data reflected selections from the Wilderness Campaign on the then US frontier. Three bad predictions come from data of single year, 1781. A talk with Helmbold suggests that these particular sets of data may be unreliable.

All 59 values of un-normalized Hi were plotted as a function of Ci/Ni (Figures 3). While it is not remarkable that the points fall along the line (because Hi is derived from Ci/Ni), their distribution is remarkable. For the attacker in Figure 3a nearly all points fall between zero and 0.4; or about where conventional wisdom predicts that the loser ceases to be a fighting force. For the defender in Figure 3b the story is told by the number of points past the peak.

[Figure, Go Here--Scatter plots of un-normalized Hi versus (Ci/Ni) with (i=a,d)]

We now turn to a consideration of the time series data which is expected ;o be more sensitive to the hypothesis that entropy is a combat outcome predictor. It Is also the data from which we could hope to extract a C2 predictor.

The NTC Exercises

The time series information from the NTC and JANUS simulations thereof was analyzed in two ways. First, a normalized cumulative entropy calculation was per formed in which the original number of attackers and defenders were used as the numerator in equation 2 for all incremental time periods, tj. Second, the entropy was computed for each successive time period using for Ni the remaining force strength at tj, which was divided into the casualties generated between ti and tj+1

Figures 4 arid 5 summarize the NTC data. They each epics plots consisting of Ha and Hd versus time, and the two entropies versus each other for cumulative and time interval sensitive computations, respectively.

[Figures 4 Go Here plots of *Hd,* Ha, and their difference versus time; and also Hd versus Ha where both are for the cumulative entropy Morn the NTC data set]

[Figures 5 Go Here--Plots of Hd, Ha, and their difference versus time; and also Hd versus Ha where both are for the time sensitive entropy from the NTC data set;

The most striking feature of the preceding plots is to be found in Figure 5a, which is interval data, where at time 330 the attacker's entropy rises suddenly and then falls. In Figure 4a the total entropy comparisons tell the same tale. These results can be interpreted as the attacker taking the initiative, and associated risk. The risk pays off for the attacker's time interval entropy again declines whsle that of the defender remains high. Figures 4b and 5b show the clear evidence of a win by the attacker as the trajectory in Figure 4b hooks back after the initiative is taken. Points have been numbered in time ordered sequence. Plots *of* entropy versus entropy have been arbitrarily normalized to the peak of cure in equation 1 by dividing by 0.37.

We may generalize the results from the Hd versus Ha plots by looking a; the different notional trajectories for the arrow *of* time in Figures 6,

(Figures 6 Go Here--Arrows of Time]

Figures 4b and 5b are not in disagreement with Figures 6.

For comparison three JANUS excursions arbitrarily numbered one, three, and six were selected. In One case ;six) the defender is the clear winner. Shown for each instance are plots ^f entropy versus times for both cumulative and time interval cases. For case one both cumulative and time interval plots of entropic space (Hd versus H3) are included. For the other two only the cumulative results are displayed

[Figures 7 Go Here—JANUS simulations of the data already presented in Figures 4 and 6]

Other JANUS runs showed more complex behavior with ail exhibiting a persistence of the engagement beyond normal break off. Such behavior may be caused by the absence of a set of computer routines which trigger force separation under combat termination conditions.

Additional searches for a signature that a side was winning or losing based on entropy were also performed. In Figure 8, for example, we show a three dimensional scatter plot of entropy for attack and defense versus time for N T C/30 minute data and for JANUS 4 produced with the aid of a computer routine that rotates multi-dimensional data. The orientations of the three axes result from experimenting with rotations that would separate the points in a three-dimensional prciection. Additional use of this program produced the results displayed later in

gores 11.

[Figure 8 Goes Here--Three dimensional depiction of the relative entropies versus time for the NTC/30 minute and JANUS 4 runs. Both are cumulative results.]

West Wall and Inchon Data

These two campaigns which lasted about one and three weeks respectively and were characterized by troops concentrations numbering 20,000 to 50,000 men per side, were a sharp departure from the very controlled combat examples just introduced. Entropy was calculated on a daily basis using reinforced figures when appropriate. Three dimensional plots of Hd versus Ha versus time are introduced because of the greater complexity to the trajectories in entropic space.

[Fig 9 Go Here--Time plot of West Wall by daily interval entropic space plot showing numbered, time ordered points]

[Figures 10 Go Here--Time plot of Inchon Operation by daily interval and also entropic space plot showing time ordered points

[Figure 11 Goes Here--Three Dimensional plots of West Wall and Inchon]

History records the Allies as victors in West Wall and for the Inchon action as well. How well do our predictions bear up? Both time and entropic battle space plots for West Wall show a victor after some undecided early, and un-reinforced, action. In fact the process appeared from the data to be accelerating at the end. As yet we have no way to measure the velocity or acceleration in the production of entropy but it appears that it would be a very sensitive indicator of which way the battle was headed. It would also accord well with earlier comments on dissipative systems as evolutionary systems. Although we have no data the time derivative of entropy dHi/dt yields the following results where both reinforcement and casualties are considered as functions of Lime (actually a difference equation might be more appropriate here, but was not used).

$$dH\,i/dt = (CiNi - CiNi\,)/Ni^2\,[1n\,(1/(Ci/Ni)) - 1] \quad (5)$$

Examination of equation (5) shows that the sign of the entropy production will indeed depend critically on the reinforcement rate (dNi/dt) which represents flora across the boundary of a dissipative system.

For Inchon data the key element is found in the daily changes In entropy With time. The defender's (NK) entropy remains clearly above that of the attacker (US). Interpretation of the plot of entropies versus each other shows no clear pattern. This actual combat data admixes lulls in the fighting and periods of reinforcements on both sides. For instance, the UN force was reinforced on day 9, and a link-up of forces occurred on day 13. This may help to explain the clusters of points which seem to indicate two phases of activity. Military records indicate three phases. This is seen more clearly in the 3D plot in Figure 11. In Figure 11, days 9 and 13 could be read as the last day in clusters (phases) l and 2. Comparison with the historical record would seem to substantiate this finding although the clustering is not so apparent from the original data.

Another Lock

In Dockery and Woodcock (1989) we discussed the use of entropy rut forward by Jumarie (1986) in an application called relativistic information theory. That work borrowed from this paper, which was then in progress, and reported on the use of a systems dynamics package called STELLA. A STELLA model employing Lanchester equations can be used to generate attrition figures from which Hi can be computed. The reader is assumed to be familiar with these equations which may be written in a general form as

$$dx/dt = -a_1.y - b_1.xy + c_1 \text{ and } dy/dt = -a_2x - b_2xy + c_2 \qquad (6)$$

where the conventional interpretation is that at, represents aimed fire, bi represents area fire, and ci represents reinforcements.

We began with a highly stylized scenario (A) which used only aimed fire with equal initial combatants (1000) on either side. Respective attrition rates were a1 = 0.25 and a2 = 0.10 per time step with b = c= 0 for both sides. Casualty production is depicted in Figure 12a. The trajectory of points in entropy space shown in figure 12b were obtained. Comparing with the results from NTC and actual combat we see that this simulation, while generating a classical curve peaking at 0.37 is also too intense. The latter is a long standing criticism of Lanchester equations; and it appears well founded. It would also explain why deterministic Lanchester equations almost never fit real data.

[Figure 12 Goes Here--Plot of trajectory in entropic battle space for simple scenario A. together with casualties with time.]

The second scenario (B) was slightly more complex. For values of he attrition coefficients a; = 0.2 and a2 = 0.1, we took unequal initial combatants of x =3C°0 and y =1000. Since our actual combat data involves reinforcement we also reinforced side x with 100 per unit time with the results in Figures 13. Despite the greatly unequal attrition coefficients, the reinforced side wins. T he reversal :of fortunes is evident in Figure 13b.

[Figures 13 Go Here--Plot of entropies and casualties with time for scenario B where side x is reinforced continually but wins eventually. Note that the time advances toward the more dense sets of points, ie. darker line]

Finally in scenario (C) we added area fire to produce the results presented in Figures 14. The initial force strengths were unequal being x=1500 and y =1000. For side x the value of al was constant at 0.15 while for y the values of a2 are 0.1 for t < 2.5 rising to 0.3 for 0.5 time units and then declining to 0.05 after t = 3.0. The x force is subject to area fire between t=2.0 and t=2.5 time units with a value of b1 = 0.001 with b2 = c = 0.

[Figures 14 Go Here--Suite showing scenario C including casualties with time, cumulative attritions and entropy space.]

The effect of the area fire is to reverse the tide of battle in favor of side y as seen in Figure 14b. Cumulative attrition plots in 14c and 14d show that even for the victor, the Lanchester Equation battle goes well beyond the peak of the -p In p curve.

The results of the three STELLA based simulations using Lanchester equations are clearly in accord with our contention that entropy is a predictor of victory and where the reversals due to additional influences are clearly -seen.

A Unifying Interpretation

The use of an entropic state space diagram whose axes are Ha and Hd is proposed. Entropy maps the arrow of time so earlier, in Figures 6 we introduced the possible trajectories of this arrow in the State space. 'vile had depicted trajectories corresponding to respective winning paths for attack and defense with the understanding that an actual battle, in which one side or the other does not totally predominate, will show features of both.

We choose now to divide the state space into four regions (I-IV) as shown in Figure 15 Note that the space i's of necessity normalized along each axis to 1/0.37 of the value of the entropy, which we remind the reader is the maximum, of the entropy curve.

[figure 1 5 Goes *Here--Entropic State* Space for Battle Outcomes]

The regions correspond to possible battle outcomes which are summarized as follows:

- Region 1: A region of low entropy production corresponding to low casualties and ambiguous outcomes or no outcome at all. Initial phases of the battle pass through this region with potential success depending upon the details of the trajectories as indicated for the figure showing the arrows) of time.
- Region Il: Here the high values for entropy of the defender coupled with low entropy for the defender indicate that the attacker wins.

• Region III: Like Region I, this is a region with ambiguous outcomes perhaps representing the high attrition part of a battle with outcomes dependent on the direction the trajectory turns toward (I or IV). Only simulated combat seem to reach this region

• Region IV: The analogue of Region II. Here the entropy production roles are reversed and the defense wins.

Power Spectra

Recently Woodcock has investigated the possibility of applying time series analysis to combat interactions. [Woodcock, 1990]. In particular he has generated power spectra. The self-same technique was applied to the NTC casualty data treated as a time series signal with the results shown in figures 16.

[Figures 16 Go Here--Power Spectral Analysis of NTC 101 minute interval data for Red and blue. The top curve is a normalized plot of the data in the time domain and below the corresponding frequency domain normalized to frequency with the highest power contribution to the signal.]

Although the density of points is marginal for this kind of analysis, Figures 16 do show a main peak which we interpret to mean ;hat the Red attack basically came in a single wave. This is in accord with field observations. The Blue defense appears to have one (or two) higher frequency cycles indicating more than one mode of response to the attack. The foregoing appears to verify that the entropy predictions done with this data are uncomplicated by multiple thrusts so that the engagement can be regarded as single attack.

When the analysis was extended to the six JANUS simulations five 0f six Red attack simulations showed a single peak on the attack. Blue simulation responses in all cases were less complex than the actual field data. For comparison we display in Figures 17 the results for JANUS 4 and JANUS 6 as being typical of the power spectra from the simulations. Five minute simulations intervals yielded 16 useable time intervals.

[Figures 17 Go Here--Power spectra transform (bottom) and associated time domain plots (top) for JANUS 4 and JANUS 6]

A next step in using an entropy predictor based on casualty data would be to analyze an engagement with different maneuver and rates of advance. Simulated information of this sort should be possibly emerging from work of Protopopescu et al. on modeling heterogeneous combat with partial differential equations as this output has been shown to match JANUS data as well (Protopopescu et al., 1988, 89).

In another approach to extracting information from time series data, an attempt was made to look for evidence of an attractor by attempting to generate a possible Poincare section with a method discussed by Stewart (1989). The technique involves plotting a time series against itself shifted by one or two time steps. We experimented with an alternative form which is the incremental Red casualty goes versus the time shifted Blue figures in a scatter plot. There are too few data points to draw any conclusion so that the equivalent figures for Red and Blue entropy were no attempted.

Implications for C2

Because casualties and casualty rates can be measured from observables t. production of entropy based on these figures can be instrumented. Since destruction of infra-structure and materiel also contribute to the breakdown of the inherent c cohesive structure required of a fighting force, it should also be possible to measure that destruction in entropic terms. Since casualty data can be not notoriously uncertain, the .hypotheses about progress of the battle should be cast in entropic terms as well. The fuzzy entropy of deLuca and Termini (1972) already introduced is suggested as a starting point.

The rate of entropy production is also singled out as another G2 indicator. In fact the coupled casualty and reinforcement rate are really what are at issue. Measurement and display of both seem close to parameterizing something which is variously described as the "tempo" of battle. The tempo is then seen to characterize not the physical rate of advance (the usual connection) but rather the rate of structural breakdown of the fighting force.

It also seems plausible to articulate a class of entropy based planning factors to govern the development and time line of either attack or defense. It may be that the "classical" notions oz requirements for numerical superiority of the attacker at numbers like two and three `o one may be related to entropy production. We *would* venture that these ratios are intended to permit the attacker to sustain high transient entropy production rates in order to gain, and hold, the initiative while still relying on a dynamic structure. The defender on the other hand, usually car, count on more fixed infrastructure, and so would have different entropy based planning factors.

Summary

We believe we have demonstrated from the limited data available to us that the hypothesis concerning casualty production as a predictor of combat is verified from both time series analysis: and also the historical record cased on final casualty counts. Moreover, casualty based entropy emerges as a major indicator for the design of future C2 systems. Entropy is shown to have weep theoretical significance for future analysis of combat as a dissipative system and for the eventual identification of attractors in combat through time series analysis.

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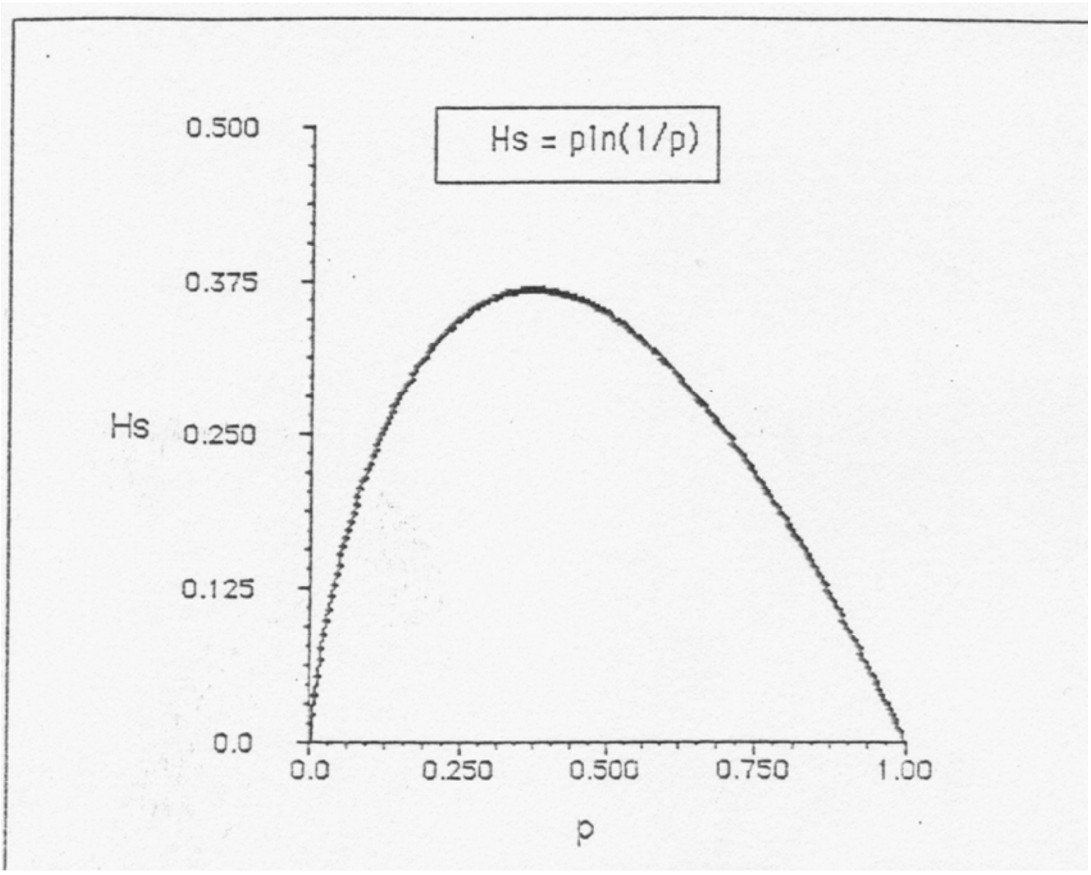

Figure 1. Graph showing the dependence of Hs on *the* value of the probability p.

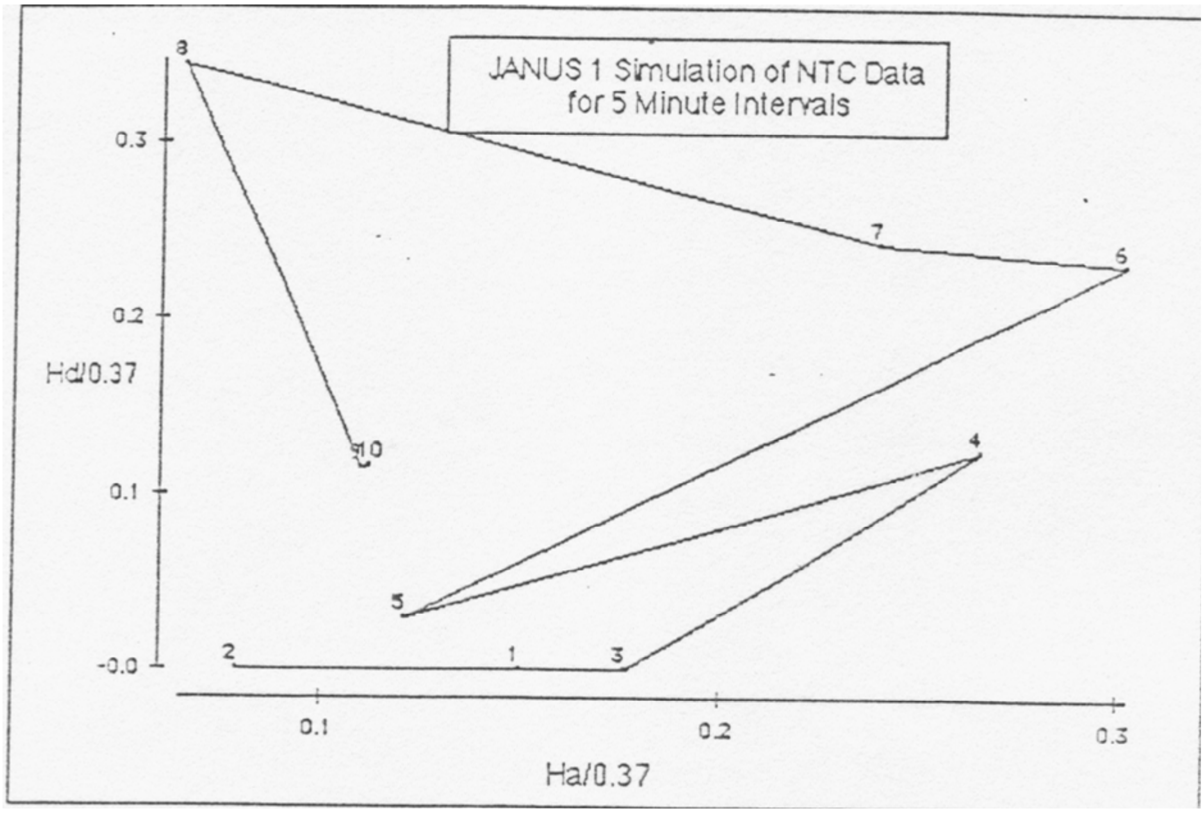

Figure 7.1d Plot of time sensitive interval data from JANUS 1 simulation data of NTC for attacker and de-fender sewing trajectory in entropic battle space.

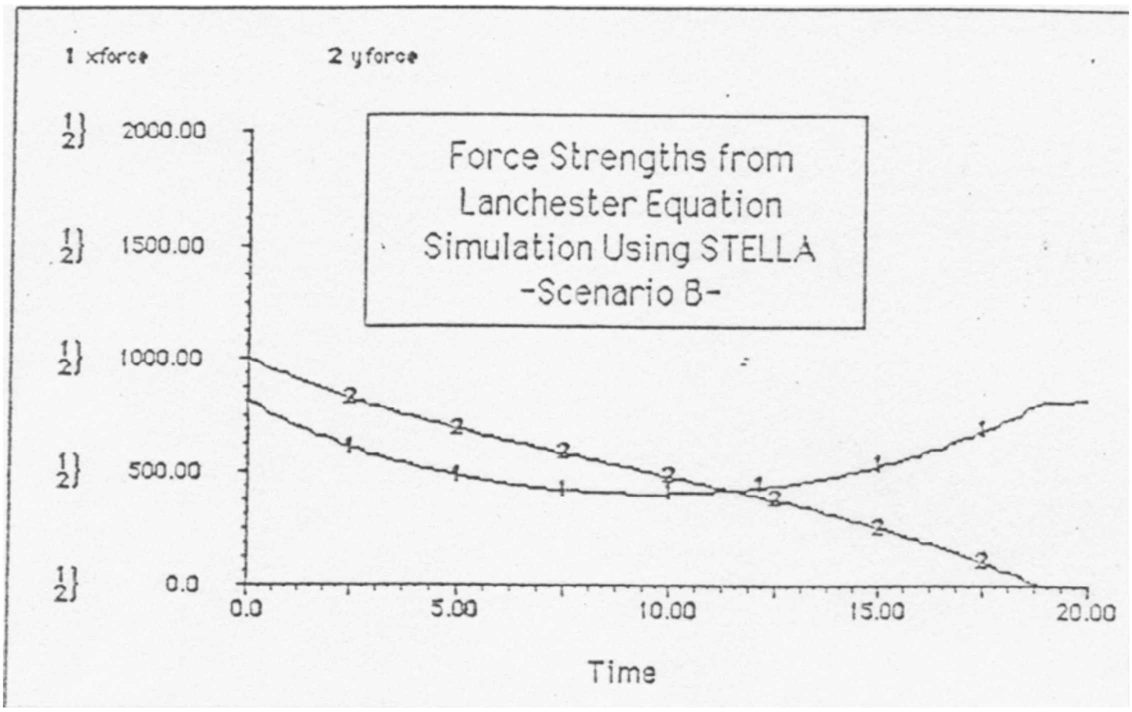

Figure 13.a: Casualty production with time for STELLA used scenario B. Unequal initial combatants and unequal attrition coefficients (respectively a1 and a2 are 0.2 and 0.1), but side x is continually reinforced at 100 units per time and eventually wins.