2.5 Using Casualty-Based Entropy to Predict Combat Outcomes

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Entropy as a measurable quantity, derivable from combat data, is introduced as a global concept by which to express the disorder of combat. Computation of entropy is performed with historical combat data Both static and time series data has been employed The use of entropy as a predictor for use in the design of command and control systems is explored. A state space diagram based on entropy values related to combat outcomes is introduced in this regard. Comparison of the entropy computations with the results of power spectral analysis of the same data is also introduced. These studies are based on a common conviction that command and control theory cannot be developed in the abstract,. divorced from a description of combat itself. That conviction is borne out in this chapter which deals with entropy computations derived from actual combat data.

Entropy Measures Provide the Basis for Assessing the Level of Combat Chaos

We have been guided in our studies of combat with embedded command and control by a number of propositions- Perhaps foremost among them has been the assumption that combat is characterized by local chaos and long range order. The local chaos is often deliberate as the goal of either combatant is to sow disorder while preserving his own structural integrity. We have hypothesized that combat with embedded command and control is a self-organizing system with training and discipline playing a major role in the aforesaid process. Thus, it is our contention that it is command and control which serves to give structure to combat. It has been almost a tenant of faith among commanders that infliction of casualties reduces the structural cohesion of a force, and in turn sows the kind of disorder that presages collapse on the battlefield. Measurement of that disorder has proven elusive although there is a theoretical quantity which is by definition a measure of disorder. That quantity is entropy.

In this chapter we shall explore the consequences for command and control of a recent proposal by Carvalho-Rodrigues (1989) that entropy, computed from casualty reports, can serve as a predictor of combat outcomes- A report of our investigation of such a possibility has been presented at a recent conference (Carvalho-Rodrigues, Dockery; and Woodcock, 1991). If this proposition can be substantiated, and we shall seek to show from historical data that it can, then entropy should prove a means through which to evaluate command and control effectivenessHaving advanced that notion, one turns automatically to the possibility that entropy may serve as a predictor when extracted from an accumulating time series. For if entropy has predictive value for command and control, then its evaluation can be instrumented from incoming reports of battlefield casualties.

Our case *will* be advanced as follows. We will begin with a necessary review of the entropy concept in a number of guises- Then the rather surprising notion is advanced that entropy is a *unifying* principle- A Shannon-type entropy equation is then tested against selected historical data as well as data derived from combat simulations and mathematical models of combat based on the Lanchester Equations. A case for the successful prediction of combat outcomes is then advanced. Finally, time series simulation data is subjected to power spectral analysis which indicates that the data is presumably from a single wave attack.

ENTROPY IS A GLOBAL *MEASURABLE* PARAMETER

It is our contention that far from being an obscure concept, entropy is a global, and measurable, parameter which is particularly appropriate for characterizing systems. In this interpretation entropy is considered as basic to the parameterization of systems as is mass to the system's physical components. Entropy is a macroscopic or extensive property. On this assumption entropy will tell us something about basic changes in the systems while ignoring details of internal systems interactions.

Measurement of overall system entropy should be as straightforward as the measurement of the mass of components for physical systems. It is not; perhaps because entropy is usually not computed- We note in passing that the general question of basic parameters for system characterization has received little attention with the possible exception of complexity parameters (Cacti, 1989)_ What becomes important to us is to pick a form of entropy which is appropriate to combat viewed as an overall system.

In a theoretical sense the concept of entropy is commonly associated in some fashion with the disorder present in a system- One usually encounters entropy for the first, and last time, in the study of thermodynamics where it enters in the expression for the second law of thermodynamics In the second law entropy (S) is related to heat (Q) and temperature (T) as $dQ/T = dS$. Maxwell's thermodynamic equations make extensive use of the entropy concept. If asked what they remember about entropy, the average engineer would probably answer that it keeps growing; and that it is not very useful. That putative conclusion reckons without the extension of the entropy concept to information theory by Shannon (1963). He expressed the equation for noise in an information channel as:

$$HS -\_p \ln p \qquad (1)$$

where p is a probability. Unlike the thermodynamic entropy S, the value of HS reaches a maximum and then declines, as illustrated in Figure 1.

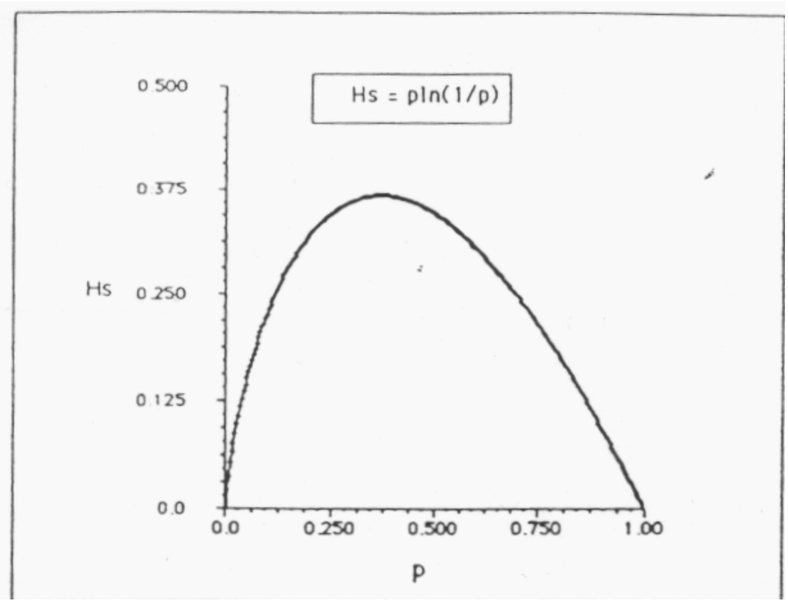


Figure 1: The dependence of the value of the Shannon Entropy HS on the value of the probability (p) in equation (1).

RELATIONSHIP OF SHANNON ENTROPY TO COMBAT

Carvalho-Rodrigues (1989) has exploited the shape of the curve in Figure 1 to predict success in simulating battle outcomes by relating casualty production to Shannon entropy values. His work assumes that casualty counts (Ci) can be related to a probability expression (Ci/Ni), where i represents either of the adversarial (arbitrarily labelled Blue or Red) forces, and where N is the force strength. Rewritten in these terms, equation (1) becomes:

$$HS = (Ci/Ni) \ln (1/(Ci/Ni)) \qquad (2)$$

In general we have that Ci = Ci(t), and likewise for N_i. Our main task in this chapter will be to test the consequences of equation (2) against actual historical combat data

What is noteworthy about the curve in Figure 1 is that it passes through a maximum and then declines. The peak is about 37'0 of p = C^. Equivalently stated, we find that although the casualty production may continue past the peak, the chosen measure of system disorder (HS) has passed its maximum It is as if the combat capability of the system described by equation (1) declines, signifying disintegration of the system itself. Other casualty production benchmarks in terms of the peak value of the curve in Figure I are approximately the following:

10 % AT 60% OF PEAK; 20% AT 84% OF PEAK; AND 25% AT 92%'o OF PEAK.

As discussed by Carvalho-Rodrigues, this kind of general behavior is in accord with the expected interaction of combat casualties and the breakup of a fighting force as a function of percent casualties sustained. A military unit with casualties of 20-30% has endured very heavy casualties indeed. If entropy is to be a valid predictor of combat outcomes, then the data should approximate a portion of such a curve as in Figure 1. For f-final outcome statistics only, the results over many battles should be so distributed. Timesensiãve data should generate only the early portions of such a curve since breakdown, end battle termination, will usually occur before the peak of the distribution.

We may note in passing that, for a dissipative system, the evolutionary criteria for the rate of generation of internal entropy production is given by the expression: dPi/dt <_ G where P, = dSi/dt (Schneider, 1988). In our investigations of combat with embedded command and control, we have argued consistently that combat is a dissipative system, and in fact may even be considered to behave as a bizarre form of ecology. Thus it begins to appears only reasonable that entropy production should finally emerge as a predictor of combat evolution, and hence a major contributor to support the command and control of combat.

OTHER ENTROPY MEASURES EXIST

Closely related to the Shannon entropy, and a generalization of it, is the following Renyi entropy expression (HG):

HG = (1/(1-G)) In (1i PiG) G > 1 (3)

where G is a kind of system gain coefficient- HG has been computed for chaotic systems. A classification scheme for such systems may be related to this property

Equation (3) does not exhaust the possible expressions for entropy. The concept of a fuzzy entropy *(HF),* expressed in the following form has been introduced by deLuca and Termini (1972) HF = h(x) + 11(1 - x) (4)

where V is a fuzzy membership function- The consequences for command and control have been discussed by Dockery (1982), where a curious property of fuzzy entropy is explored.

The property in question predicts that past a certain point HF can only be lowered by reformulating the hypothesis for which the fuzzy entropy has been calculated. The consequences for command and control are almost obvious. There are predicted to be times when collecting more information about a particular hypothesis fails to produce additional clarification! The foregoing suggests that a hypothesis about whether a side is winning or losing, based on Shannon entropy and containing fuzzy data, must be modified by the implications of equation (4).

In a recent text Ruelle (1989) ties the concept of entropy to the currently active topic o1 chaotic evolution in a manner which treats the statistical analysis of time series for deterministic non-linear systems. While we do not believe for the moment that combat is necessarily deterministic, such properties may be associated with individual attrition processes. Moreover, we have suggested in our analysis of combat with embedded command and control that combat is a chaotic dynamical system of great complexity Ruelle treats the Kolmogrov-Sinai invariant in chaotic dynamics. This invariant measure the asymptotic rate of information production, and is identified with entropy. Information is created as a system evolves. In our case we have system devolution corresponding to the creation of casualties. Connection of entropy with a system invariant is in accord with our previous remarks about entropy as an analogue for a combat system of the physical parameters like mass associated with physical systems.

Still other connections may be made. We have used cellular automata to model combat and have successfully fitted a Lanchester equation model to the data generated by a series of automata simulations. Appropriate entropy values for such cellular automata-generated data may be defined, and this is the subject of on-going investigations. Casti (1989) has introduced a "topological" entropy, which is defined as a measure of the likelihood of a particular sequence of cells will be produced when starting from a random initial configuration. A companion "measure" entropy is also introduced to give the probability that one of the configuration possibilities available under topological entropy will actually occur- We conclude from this that a further formalism may be available for estimating entropy of combat when modeled with cellular automata A discussion of spatial and temporal entropies related to the representation of combat entities as cellular automata has been provided by Woodcock, Cobb, and DePace (1989).

Combat-Related Casualty Data

We turn now to a description of our investigation of combat-related entropy by introducing the data sets that we used in our analyses. Computations of values of the Shannon entropy (HS) were performed on the following types of combat-related data:

- ***Time-independent (or combat outcome) casualty data,*** which consisted of profile data on battles for which the only casualty information consists of a tally of the initial forces and total casualties on each side, but not the temporal variations in force strengths during the battle.
- ***Time-dependent casualty data,*** which was obtained from several sources including: the historical record; a field exercise and a JANUS simulation of that exercise; and a Lanchester equation-based combat simulation

TIME INDEPENDENT COMBAT OUTCOME

We have used data derived from a series of historical battles that has been assembled by Dupuy for Helmbold of the US Army Concepts Analysis Agency (Dupuy and HERO, 1986; Helmbold, 1986) in order to test the ability of the Shannon entropy measure (HS) to predict combat outcomes. HeImbold (1987) has considered the question of a link between casualties and victory and was also the source of many insightful comments during our investigations (Helmbold, 1989).

The basic data set contains detailed historical descriptions of some 601 battles from circa 1600 until circa 1970. Data on straight forward conflict without excessive maneuver, and preferably in a single assault or meeting engagement was desired It was hypothesized that if the HS for the adversarial forces was a combat outcome predictor then it would probably have the best chance to manifest itself under the least complicated ground combat conditions- Historical battles without complex command and control were presumed to fit such a description. Therefore, not wishing to introduce additional complexity, the following selection criteria were set which yielded 59 battles satisfying the following criteria:

Under 10,000 combatants per side.
Battles which lasted up to ten hours.

The data set gives only the final casualty tallies. The list is presented in Appendix A. A few selections from this data set are displayed in Table 1.

Table 1: Sample battles used to compute Shannon Entropy values (H_S).

BATTLE	DURATION (HOURS)	CASUALTY RATIO: ATTACKING FORCE	CASUALTY RATIO: DEFENDING FORCE	OUTCOME
Culloden (1746)	0.67	1558/5400	309/9000	Defense
San Jacinto (1836)	0.30	39/743	1600/1600	Attack
Hill 272 (1918)	2.5	109/2950	250/2563	Attack
Rawiyeh (1967)	4.0	150/5350	300/4350	Attack

This data subset consisted of results in which the attackers were favored by a ratio of better than two to one. Three cases that resulted in draws were arbitrarily assigned to the nominal defender. In summary=

Mode	Number of Cases
Attack	38
Defense	18
Draw	3

The battles are shown identified by century below with a further breakout for World Wars I and II. The essential absence of battles from WWII is a consequence of our selection criteria

1600s	1700s	1800s	1900s	World War I	World War II
4	13	12	30	23	1

The duration of combat activities fell within the following ranges for the sample data set:

0-1 hr.	> 1-2 hr.	> 2-4 hr.	$\geq$ 4 hr.
13	10	18	18

No further profiling of these actual historical combat data was undertaken during our studyWe will now discuss the sources of time-dependent casualty data

TIME DEPENDENT COMBAT-RELATED DATA

Obtaining time dependent information on casualty production proved to be a more elusive task. Our primary input came from some data released to us by the National Training Center (NTC) located at Fort Irwin, California, at which military exercises are conducted (Ingber, 1989). Laser firings substitute for live ammunition during such simulated exercises- Conditions strongly favor the attacker, who represents a semi-permanent on-site aggressor force

One set of time profiles of casualties in terms of vehicles destroyed were made available in 5, 10, and 30 minute intervals. The last set was chosen for the most extensive analysis. Smaller time-interval data sets were used in other related analyses. Computations of the accumulating value of the entropy were based on the initial combat conditions. In addition, the entropy generated in any 5 and 30 minute interval was also computed- The latter was to prove most illuminating as it gave us a more synoptic profile of the mutual attacker and defense responses. Anticipating results, yet to be introduced, it was observed that the attacker's entropy rises suddenly as the attack is pressed then must decline rapidly if the attack is to succeed. The unsuccessful defense signature, by contrast, is a rising entropy that never falls back to low levels signalling defeat.

Excursions to the basic exercise scenario are generated by running through the JANUS simulation, which was originally developed at Lawrence Livermore Laboratories. Six such excursions were also provided in five minute time steps- We used these as well but found they showed somewhat different properties for the engagement process chiefly in terms of the persistence of firings by the defender force after its defeat.

The second sort of time series data came from the actual combat activities associated with Operation West Wall that took place near Aachen in early 1945. This campaign was directed at the Siegfried Line and was characterized by heavy Allied reinforcements during the course of the battle- The data, and that for the Inchon campaign discussed below, together with commentary on both campaigns, was graciously supplied to us by Helmbold (1989). Use of information from an actual battle introduces several davs of data sometimes with the additional complexity of reinforcements and maneuver It is necessary to track the battle for a longer time span as the kind of detail available from NTC is not recorded (or recordable) from actual engagements- Daily causality figures from the actual battles were converted to entropy equivalents.

The most complex data used was from a campaign lasting some 20-21 days (September y to October 4) after the United Nations landing at Inchon (North Korea) in 1950 during the Korean war- In this case heavy reinforcements characterized the North Korean side, but the United Nations side was reinforced as well at day 9-10 of the campaign. Entropy was computed from daily casualty figures taking into account reinforcements during the subject period.

Our final source of time-series casualty data, or rather pseudo-data, came from a simulation developed especially for this work- Basically we generated a time history of mutual attrition for two sides using Lanchester equations in a manner to be described later on. It was anticipated that the "pure" attrition-based Lanchester solution would provide an idealized data sample. And, as we shall see, we were not disappointed

The Analysis of Combat-Derived Data

HISTORICAL BATTLES

We computed entropy values Hd and Ha (for defender (d) and attacker (a), respectively) using equation (2), but with casualty production normalized to unit time, for all 59 battles in the data setThis provided what was fundamentally an averaged entropy production rate However, a couple of very short battles gave anomalously high entropy rates using this technique

From the computed entropies the quantity ô; _ (Hd - Ha) was selected as the predictor of the combat outcome. Results were as follows in Table 2a:

Table: 2a: Results of normalized computation of δ_i

Mode	Correct Prediction	Incorrect Prediction
Attack	35	5
Defense	15	6
Totals	50	9

The results in Table 2a were compared with the hypothesis that the figures could have been generated at random. The X2 value was computed with one degree of freedom for a 2x2 contingency table with a value of X2 = 25.75. (It should be noted that the displays in Tables 2a and 2b are not in the proper form for a contingency test. The rows must be relabeled and the numbers in the second row reversed to use the standard X2 formula.) By comparison the significance level for 95% confidence is 3.84 and for 99% confidence level is 6.64

As we have said the values for Hd or Ha above are based on an entropy which has been "normalized" to unit time. This is equivalent to some kind of average entropy production. Results were somewhat less sanguine when the parameter &i' = (Hd - Ha) was computed from un-normalized entropy data. The results of this investigation are presented in Table 2b:

Table: 2b: Results of normalized computation of δ_i'

Mode	Correct Prediction	Incorrect Prediction
Attack	31	7
Defense	15	6
Totals	46	13

The X2 value for Table 2b was X2 = 16.25 which is still above the 99% confidence level.

In analyzing the data we discovered that real problems with the predictor can arise for battles with casualties which go on beyond the 37% stage, which is the peak of the curve generated by the relationship (- p In p). This is because the curve has zero values at both ends- Thus, an attacking force, which lost but a percent or two of its force, but annihilated the defender would incorrectly be predicted as the loser. To compensate for this phenomenon we also computed a function f(Hi) (where the coefficient (i) represents (d) and (a), the defender and attacker, respectively) from the un-normalized area under the curve in Figure 1. Thus:

f(Hi) = Z; = fp In (1/p) dp = (p2 /2) (In (I /p) + 1/2) (4)

Table 2c depicts these results for the quantity of = (Zd - Za):

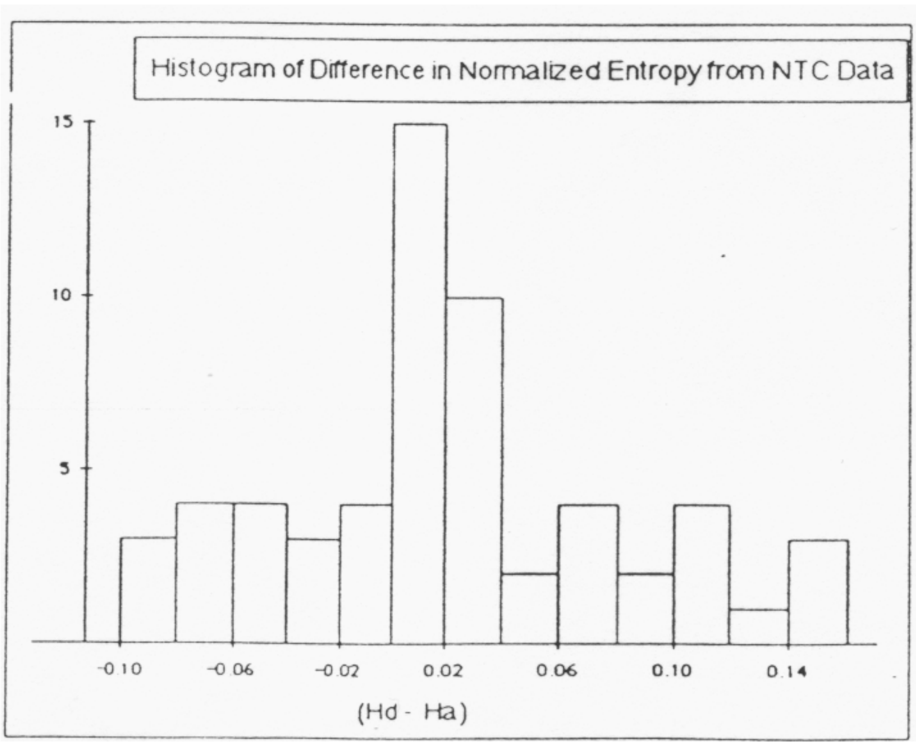
Table 2c Results of un-normalized computation of *Ai*

Mode	Correct Prediction	Incorrect Prediction
Attack	34	4
Defense	16	5
Totals	50	9

The *XZ* for Table 2c was X2 = 26.03, the best of the three values by a small margin.

Whatever results we get from use of any of the three predictors in Tables 2a, 2b, and 2c, we are faced with the fact that the predictive value of entropy lies to the left side of the peak value of the (- p In p) curve- This conclusion will be substantiated in Figures 3 which are presented below. It is to be noted that small differences in attacker and defender entropy predominate in the normalized case, as shown in Figures 2, where histograms for &i, &i', and of are displayed.

The predictions were better for battles after about 1850. Some of the early combatderived data
reflected selections from the Wilderness Campaign on the then United States frontier. Three bad
li predictions come from data of single year, 1781. A talk with Helmbold suggested that these particular
sets of data may be unreliable. All 59 values of unnormalized H; were plotted as a function of Ci/Ni
for the attacking and defending forces (Figures 3a and b). While it is not remarkable that the points fall
along the line (because H; is derived from Ci/Ni), their distribution is. For the attacker in Figure 3a
91 nearly all points fall between zero and 0.4, or about where conventional military wisdom predicts that
the loser ceases to be a fighting force. For the defender in Figure 3b the story is told by the number of
points located past the peak



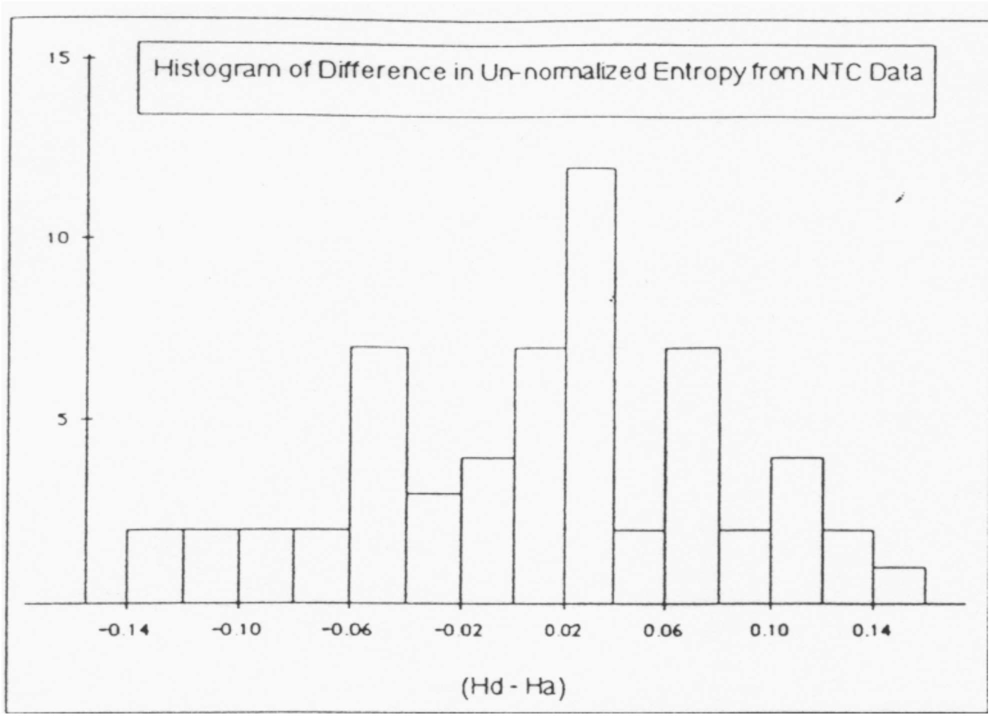


Figure 2b.

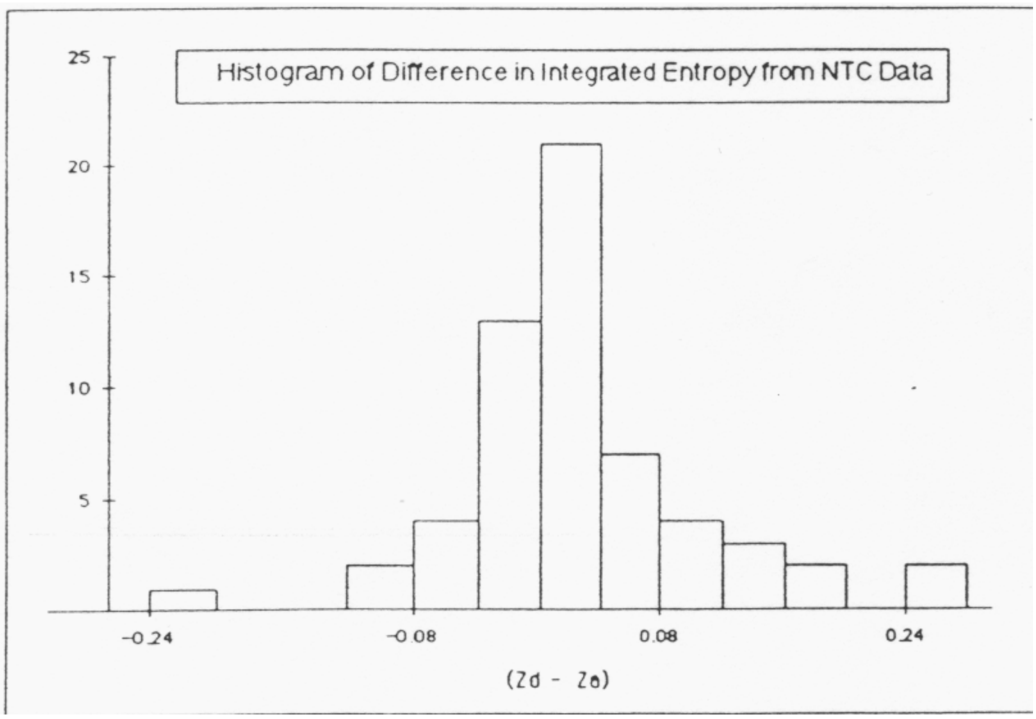


Figure 2c.

Figures 2. Histograms of the normalized quantity &i = (I-Id - Ha) (Figure 2a) the un-normalized quantity bi' = (Hd - Ha) (Figure 2b) and the unnormalized quantity o; = (Zd - Za) (Figure 2c) from Tables 2a, 2b, and 2c, respectively.

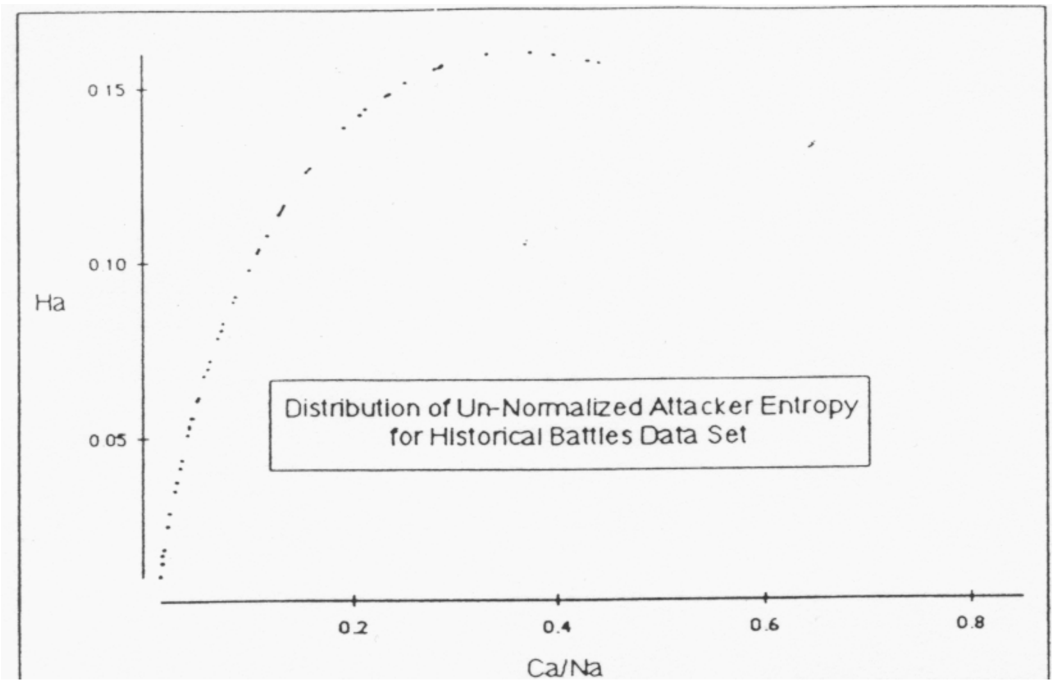


Figure 3a.

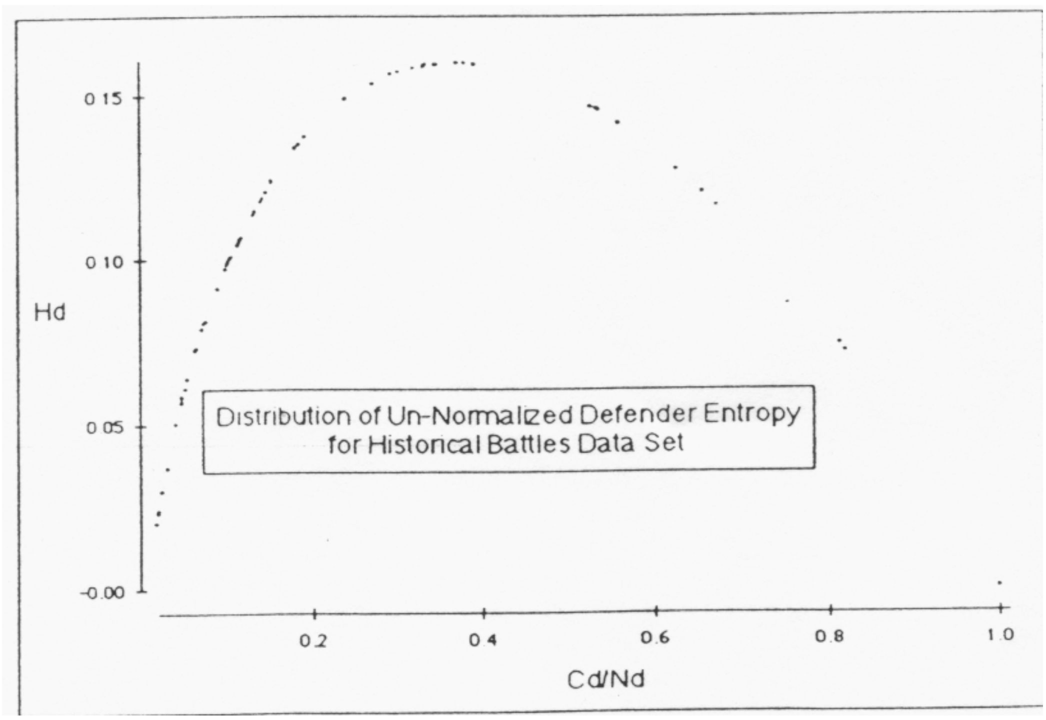


Figure 3b.

Figures 3: Distribution of entropy for 59 historical battles along the curve given by (p In p) for the attacker (Figure 3a) and defender (Figure 3b).

We now turn to a consideration of the time series data which is expected to be more sensitive to the hypothesis that entropy can serve as a predictor of combat outcomes. It is also the data from which we could hope to extract a command and control predictor.

THE NATIONAL **TRAINING** CENTER EXERCISE-DERIVED DATA

The time series information from the National Training Center (NTC) and JANUS simulations thereof was analyzed in two ways. First, a normalized cumulative entropy calculation was performed in which the original number of attackers and defenders were used as the numerator in equation (2) for all incremental time periods, tj. Second, the entropy was computed for each successive time period using for Ni the remaining force strength at tj, which was divided into the casualties generated between tj and tj+1 The NTC data is summarized in Figures 4 and 5. These figures each present plots consisting of Ha and Hd versus tune, and the two entropies versus each other in a form of phase portrait (which we have called the *Entropic Space)* for cumulative and time-interval sensitive computations, respectively.

The most striking feature of the preceding plots is to be found in Figure 5a, which is generated from time-interval data, where at time 330 the attacker's entropy rises suddenly and then falls. In Figure 4a the total entropy comparisons tell the same tale. These results can be interpreted as the attacker taking the initiative, and associated risk- The risk pays off for the attacker's time interval entropy again declines while that of the defender remains high. Figures 4b and 5b show the clear evidence of a win by the attacker as the trajectory in Figure 4b curves back after the initiative is taken- Points have been numbered in time ordered sequence. Plots of entropy versus entropy have been arbitrarily normalized to the peak of curve in equation (1) by dividing by 0.37. We tray generalize the results from the Hd versus I la plots by looking at the different notional trajectories for the "arrow of time" in Figures 6a and bb.

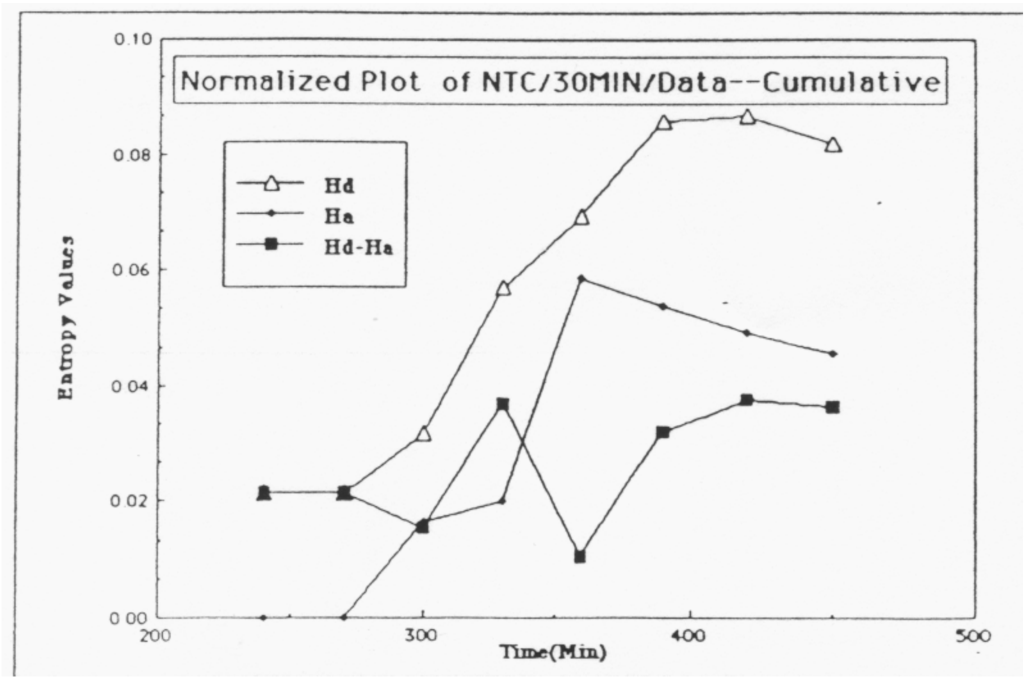


Figure 4a: Plots of NTC 30-minute cumulative data for attacker (Ha) and defender (lid) entropy, and their difference (Ha - Hd) as functions of time.

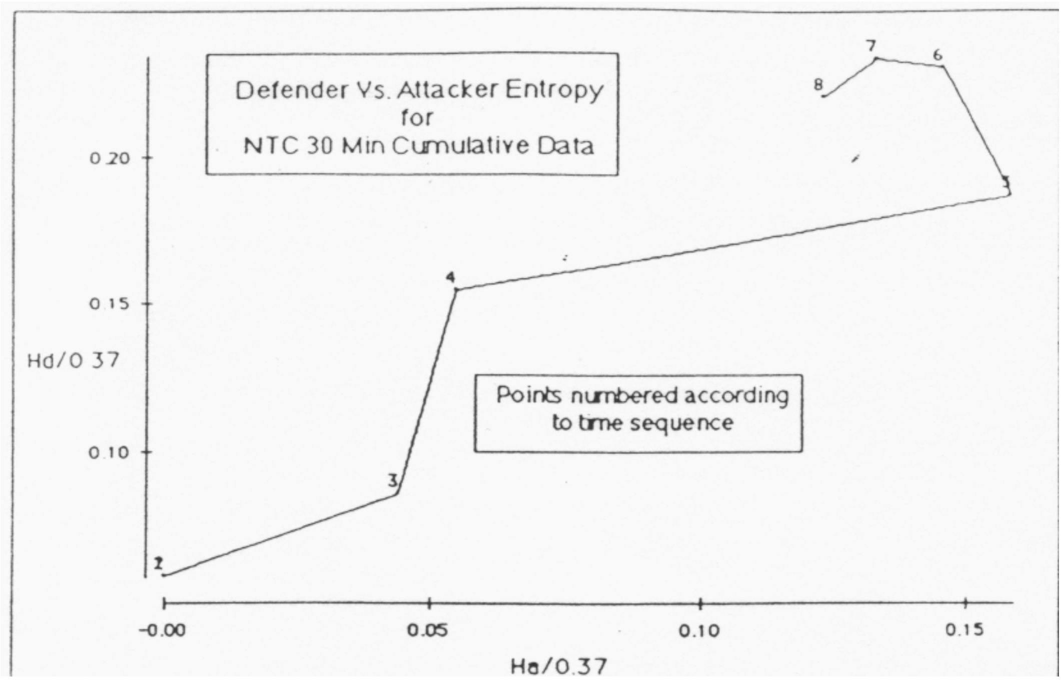


Figure 4b: Plots of NTC 30-minute cumulative data for simultaneous attacker (Ha) and defender (Hd) entropies each normalized by dividing by 0.37.

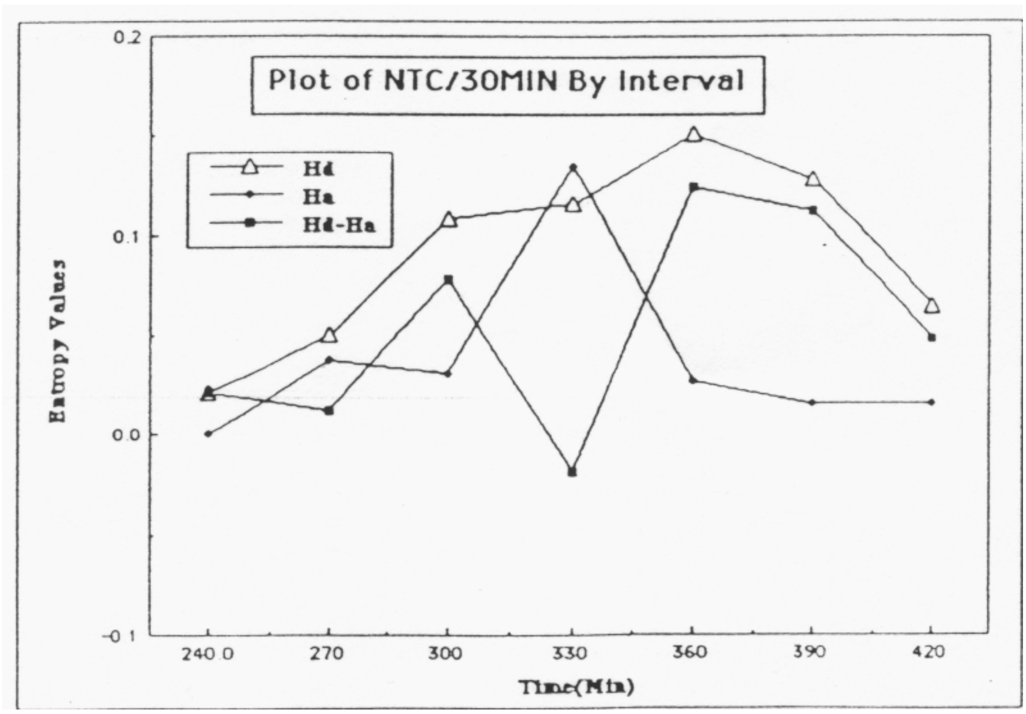


Figure 5a: Plots of NTC 30-minute cumulative data for attacker (Ha) and defender (lid) entropy, and their difference (Ha - Hd) as functions of time.

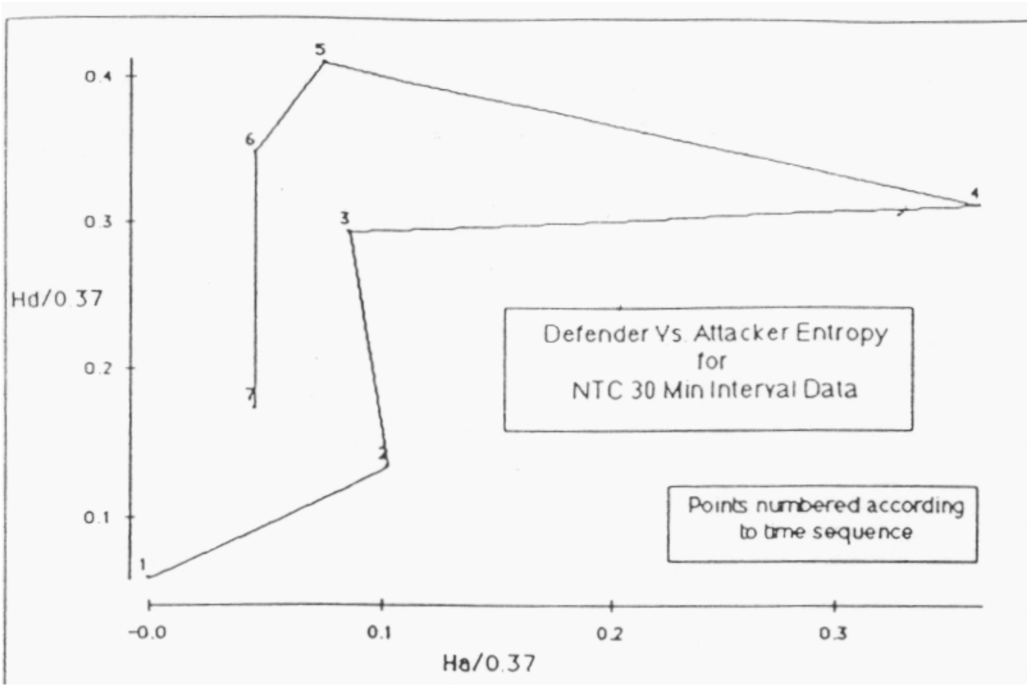


Figure 5b: Plots of NTC 30-minute cumulative data for simultaneous attacker (Ha) and defender (Hd) entropies each normalized by dividing by 0.37.

For comparison we have computed entropy values for data derived from three JANUS simulations, arbitrarily numbered one, three, and six. In one case (six) the defender is the clear winner, while in four the attacker is the clearcut winner. Shown for each instance are plots of entropy as a function of time for both cumulative and time interval cases- For each case both cumulative and time-interval plots are presented plus the entropic space (H-d versus ***Ha)*** are presented (JANUS 1 _ Figures 7a, b, and c; JANUS 3: Figures 8a, b, and c; 1 ANUS 6: Figures 9a, b, and c)_

Other JANUS runs showed more complex behavior with all exhibiting a persistence of the engagement beyond normal break-off described in the figures presented in this chapter. It is as if the simulations "didn't know" when to quit This persistence may be due to the imposed structure of the computer code and the absence of a formal structure for force dissolution and combat termination.

Additional searches for a signature that a side was winning or losing based on entropy were also performed. In Figure 10, for example, we show a three dimensional scatter plot of entropy for attack and defense versus time for NTC/30 minute data and for JANUS 4 produced with the aid of a computer routine that rotates multi -dimensional data. The orientations of the three axes result from experimenting with rotations that would separate the points in a three-dimensional projection onto a two-dimensional plane. This approach was also used to produce the results displayed in Figures 13.

Use of three-dimensional graphical displays of combat-related entropies makes it possible to identify related episodes of combat activity since such episodes appear to form aggregates when plotted in sequence in three-dimensions. This technique will provide an insight into aspects of the entropic-structure of combat.

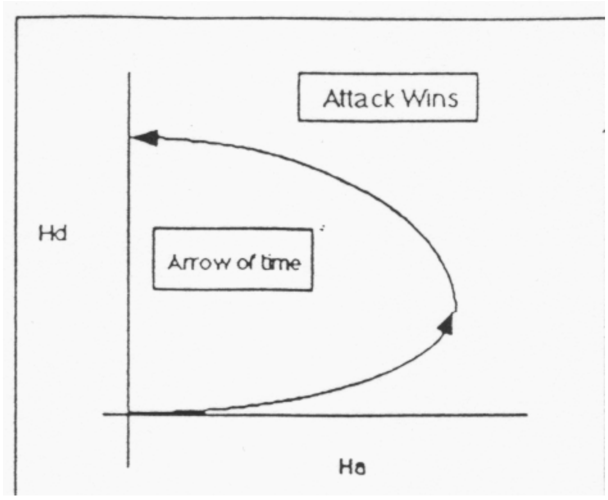


Figure 6a.

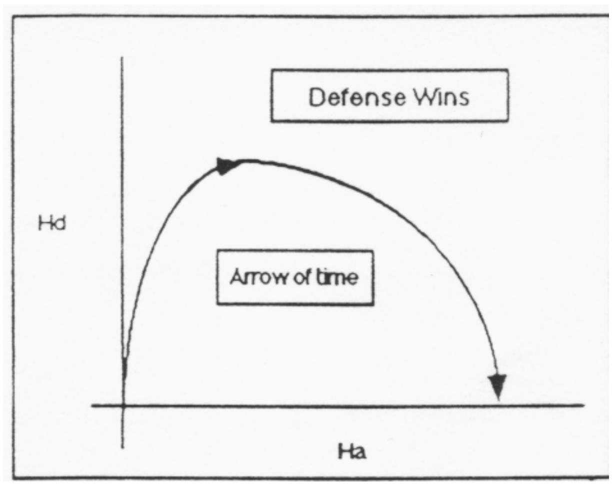


Figure 6b.

Figures 6: The "Arrows of Time" showing notional entropic space plots for an attacker victory (Figure 6a) and a defender victory (Figure 6b).

WEST WALL AND INCHON COMBAT-DERIVED DATA

Having analyzed data derived from the National Training Center, and related combat simulations, we now turn to an analysis of data derived from actual combat engagements: the West Wall campaign during World War II and the Inchon campaign during the Korean War.

These campaigns lasted about one and three weeks respectively, and were characterized by troop concentrations in the ranges of some 20,000 to 50,000 men per side, and were a sharp departure from the very controlled combat examples just introduced Entropy was calculated on a daily basis using reinforced force strength figures when appropriate. Threedimensional plots of Hd versus Ha versus time are introduced because of the greater complexity of the combat trajectories in entropic space.

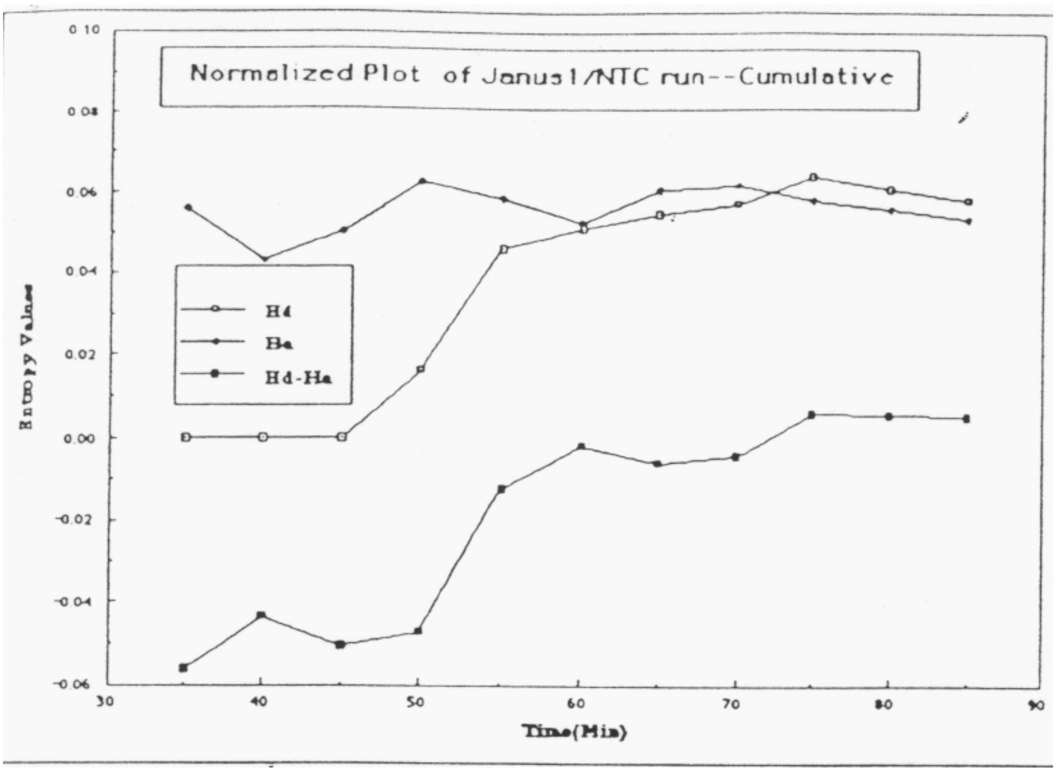


Figure 7a: Cumulative plot of JANUS 1 simulation of NTC data

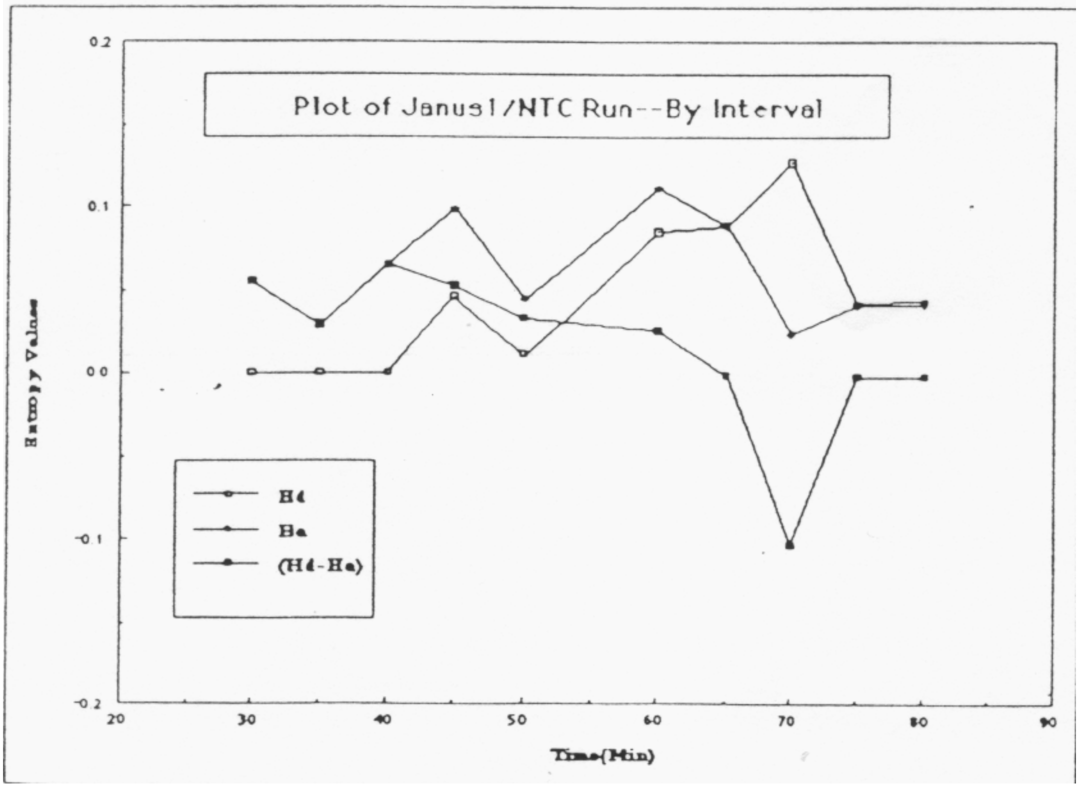


Figure 7b: Time interval plot of JANUS 1 simulation of NTC data

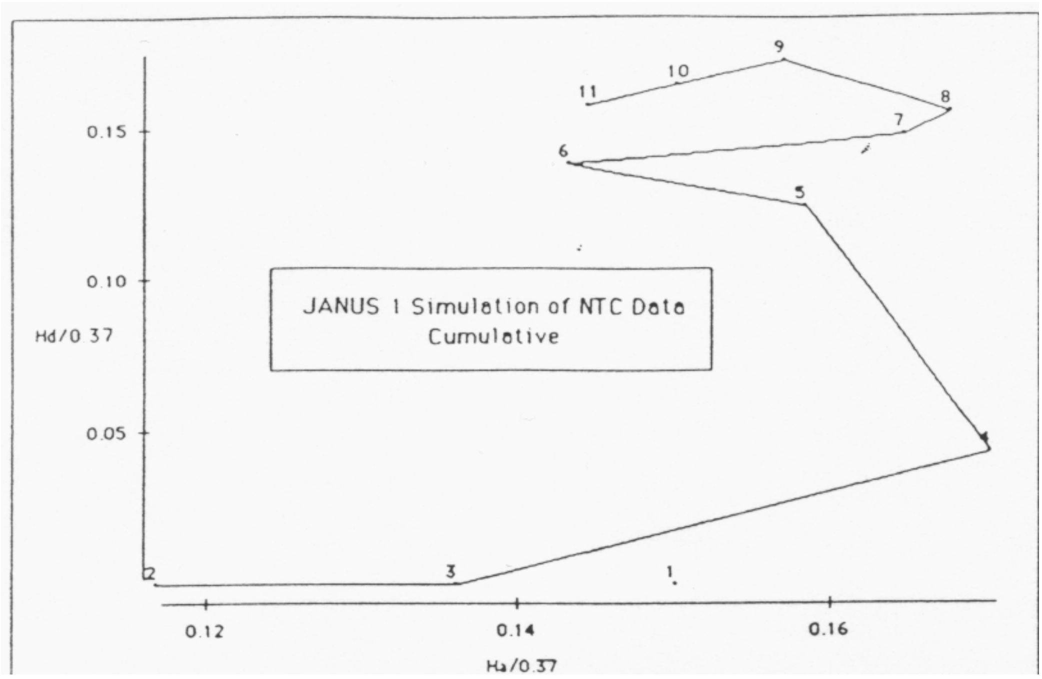


Figure 7c: Entropic space plot of JAN-US 1 simulation of the NTC data
Figures 7.= JANUS 1 simulation of the NTC data showing cumulative (Figure 7a), time interval (Figure 7b), and entropic space (Figure 7c) plots

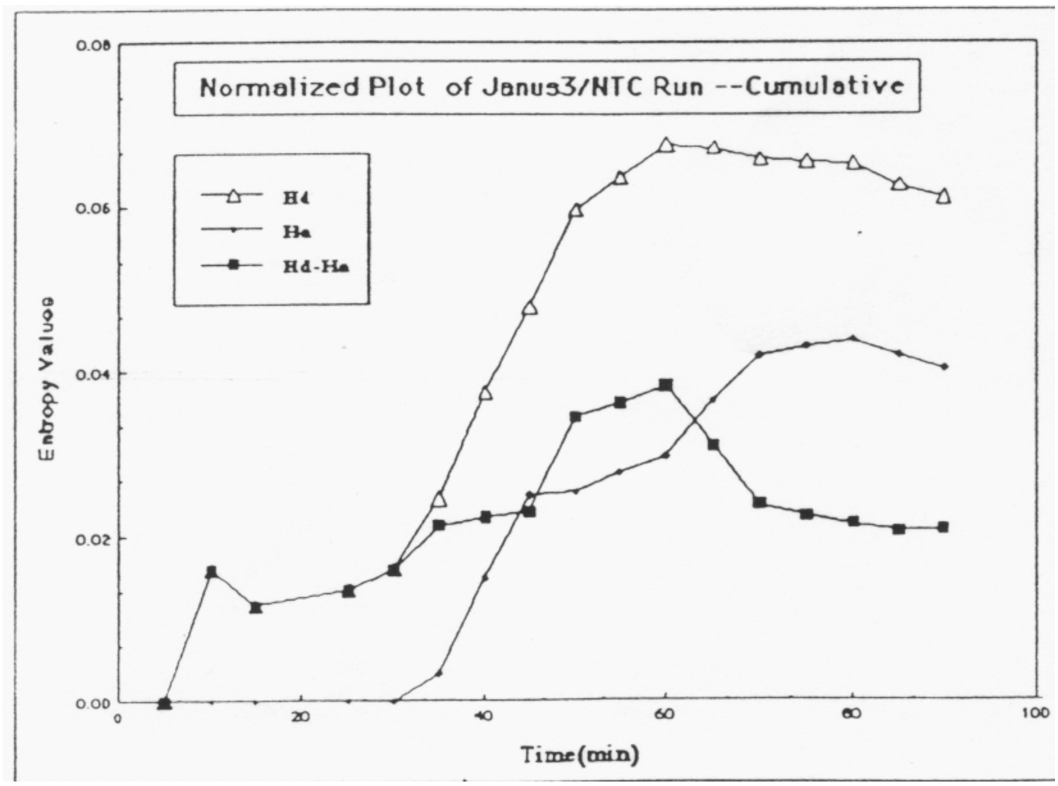


Figure 8a: Cumulative plot of JANUS 3 simulation of NTC data

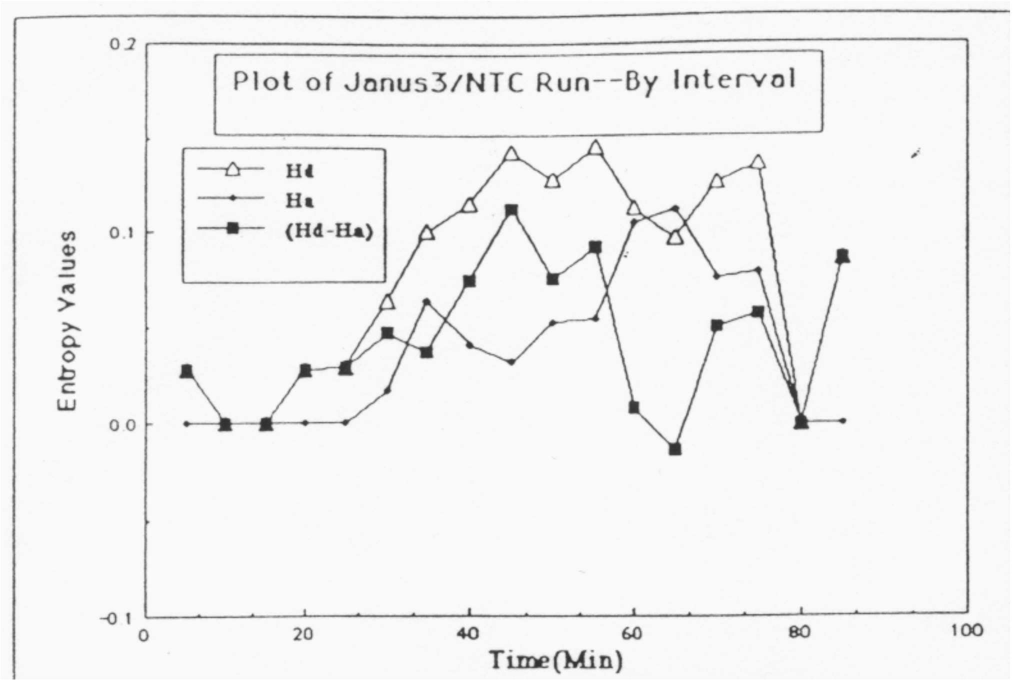


Figure 8b: Time interval plot of JANUS 3 simulation of NTC data

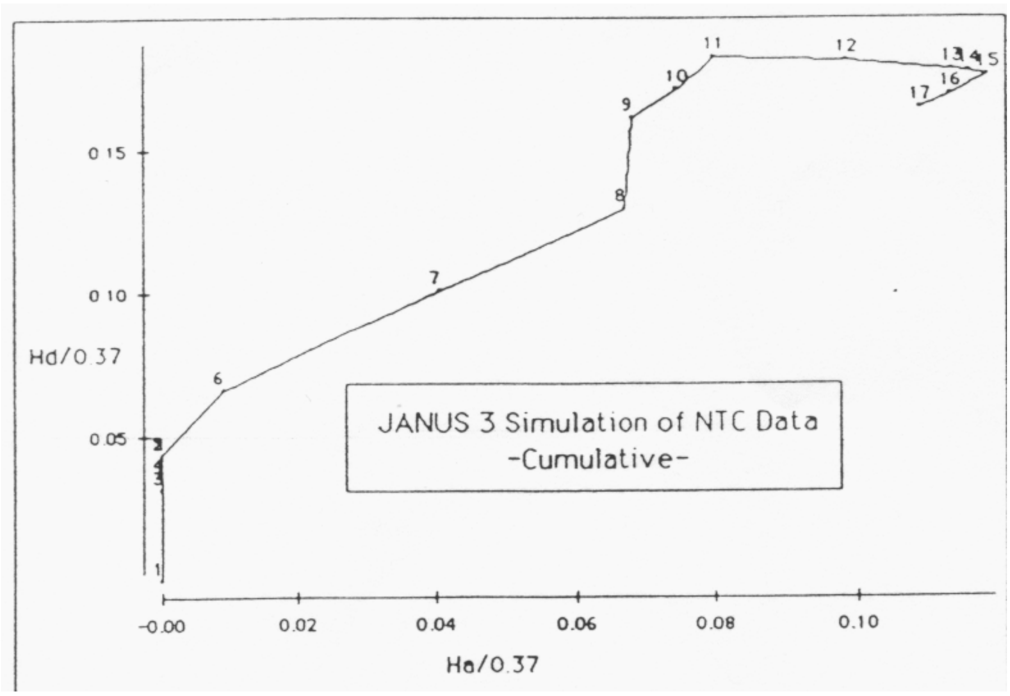


Figure 8c: Entropic space plot of JAN-US 3 simulation of NTC data
Figures 8: JANUS 3 simulation of the NTC data showing cumulative (Figure 8a), time interval (Figure 8b), and entropic space (Figure 8c) plots.

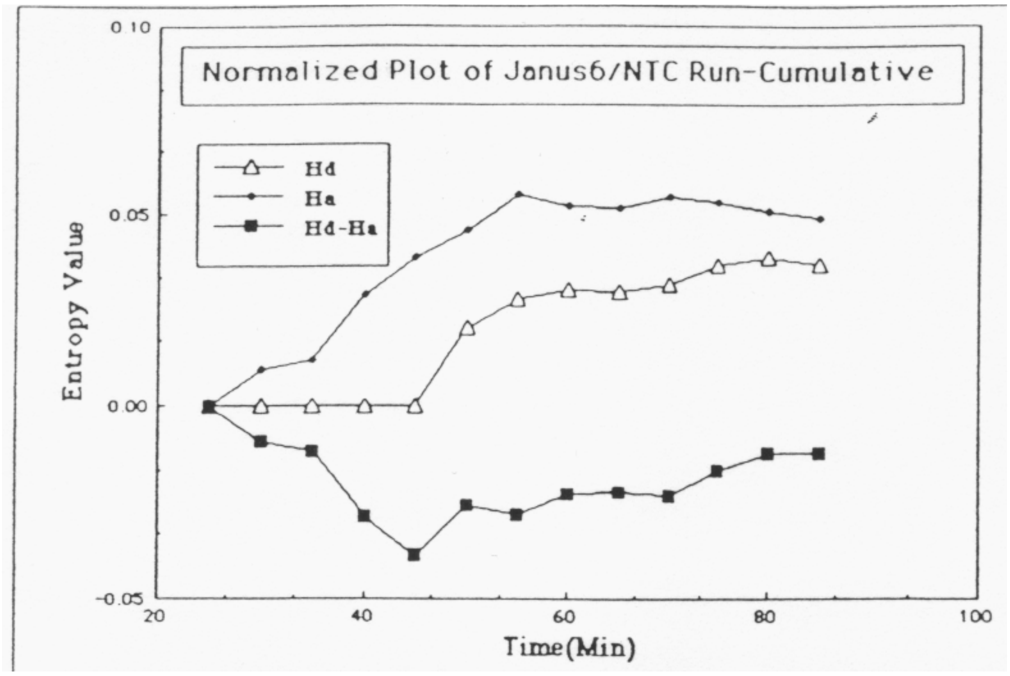


Figure 9a: Cumulative plot of JANUS 6 simulation of NTC data -

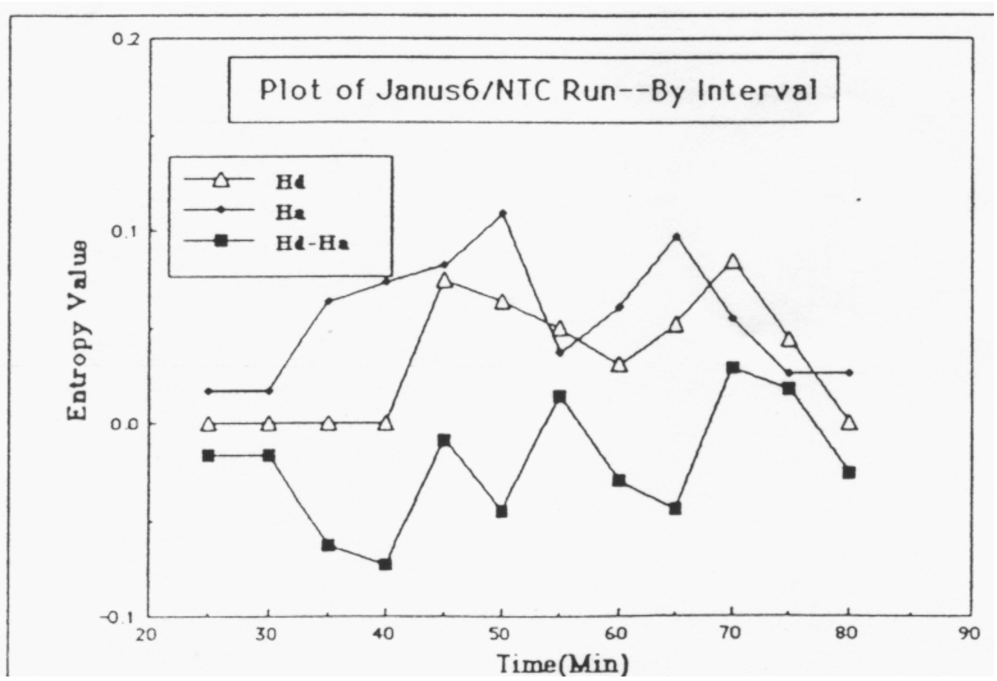


Figure 9b: Time interval plot of JANUS 6 simulation of NTC data.

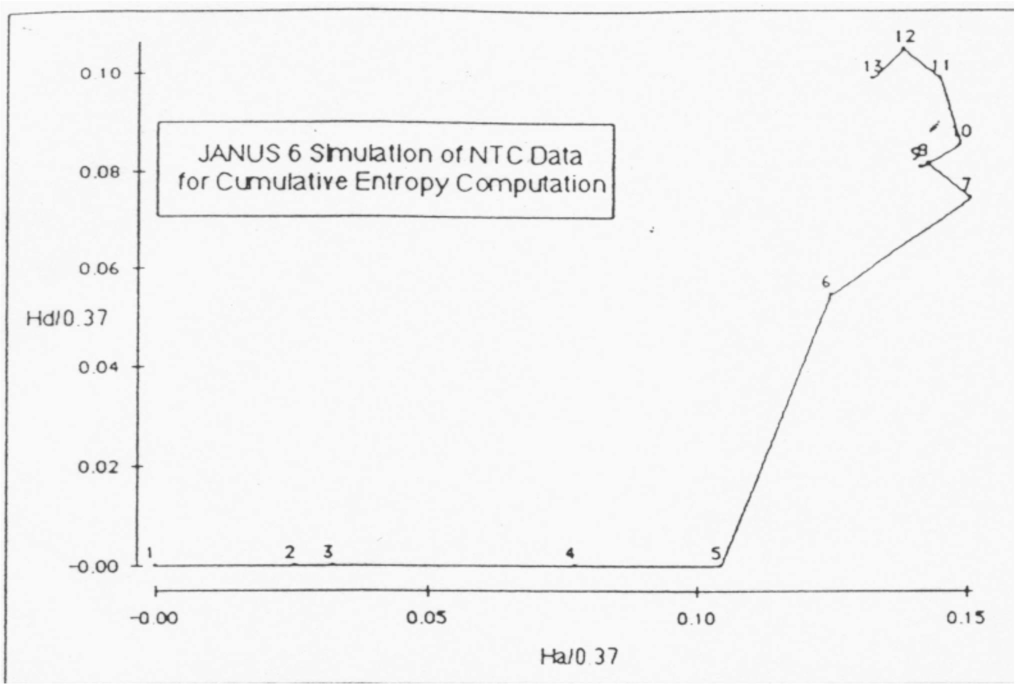


Figure 9c: Entropic space plot of JANUS 6 simulation of NTC data
Figures 9: JANUS 6 simulations of the NTC data showing cumulative (Figure 9a), time interval (Figure 9b), and entropic space (Figure 9c) plots

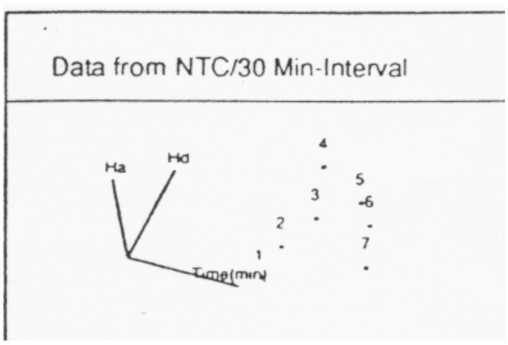


Figure 10a: Three-dimensional plot of attacker (Ha) and defender (Hd) entropies vs. tune for NTC-30 minute data

History records the Allies as victors in the West Wall and the Inchon campaign as wellHow well do our predictions bear up? Both time (Figure 11 a), entropic space plots (Figure I 1 b), and the three-dimensional time-ordered entropy plots (Figure 1 I c) for the West Wall show a victor after some undecided early, and unreinforced, action. In fact the process appeared from the data to be accelerating at the end- As yet we have no way to measure the velocity or acceleration in the production of entropy but it appears that it would be a very sensitive indicator of which way the battle was headed. It would also accord well with earlier comments on dissipative systems as evolutionary systems. Although we have no data, the time derivative of entropy dHi/dt yields the following results, where both